

AMAT 307 FINAL EXAMINATION 2005 ANSWERS

1.

$$(a) \frac{t^2}{5} + \frac{c}{t^3}, \quad (b) e^{xy} + x^2y + \frac{y^2}{2} = c$$

(c)  $y = 1$  is unstable,  $y = 5$  is stable,  $y = 6$  is unstable.

$$(d) y = c_1e^{-t} + c_2e^{-2t} - te^{-2t}$$

$$(e) W = \frac{3}{t+1}$$

$$(f) y = c_1 \cos(2t) + c_2 \sin(2t) + c_3t \cos(2t) + c_4t \sin(2t)$$

$$(g) R = \infty, \quad a_n = 2^n/n!$$

(h) Converges for  $-1 \leq x \leq 5$ , divergent at  $x = -2$

$$(i) \frac{6e^{2-2s}}{(s-1)^4}$$

$$(j) 2te^{-3t}$$

$$(k) 2e^t \cos t + 3e^t \sin t$$

(l)  $\mathbf{y} = c_1e^t[1, 1, 1]^t + c_2e^{-2t}[1, -1, 0]^t + c_3e^{-2t}[1, 0, -1]^t$ , where  $[a, b, c]^t$  is to be interpreted as a column vector.

$$(m) \mathbf{y} = 2e^t[1, 1]^t + e^{3t}[-1, 1]^t$$

$$(n) \mathbf{y} = c_1e^{2t}[1, 0]^t + c_2e^{3t}[1, 1]^t$$

$$(o) \mathbf{y} = c_1e^{2t}[\sin t, \cos t]^t + c_2e^{2t}[-\cos t, \sin t]^t$$

2.

$$y = \frac{140,000}{\ln 2} + (200,000 - \frac{140,000}{\ln 2})2^{t/7}$$

3.  $y = 2t^2 \ln t + 16t^2$

4.

$$y = e^{2t}(c_1 \cos t + c_2 \sin t) + \frac{\sin 2t}{65} + \frac{8 \cos 2t}{65} + \frac{5t^2}{8} + t + \frac{11}{20}$$

5.

$$y = \frac{c_1}{x} + c_2x^2 - \frac{\ln x}{3x} - \frac{1}{9x}$$

6.  $y = 2 + x + x^2 + \frac{x^3}{3} + \dots$

7.

$$a_{n+2} = \frac{(n-2)(n-3)}{(n+2)(n+1)} a_n$$

8.  $y = e^{-t}(2t^2 + t + 2)$

9.  $\mathbf{y} = c_1 e^{3t} [1 - t, t]^t + c_2 e^{3t} [-t, 1 + t]^t$