AMAT 307   MAPLE ASSIGNMENT   Fall 2005
Due Dec.8

ASSIGNMENT

1. Find the area bounded by $x = 0$, $x = 4$, $y = 0$, $y = 1/(1 + x^6)$. Give both the exact answer and the numerical answer (numerical ans. $= 1.047002..$).

2. Determine the general solution of $t \cdot \cos t \cdot y' + (t + 2) \sin y = 0$.

3. A first order DE of the form $y' = q_1(t) + q_2(t)y + q_3(t)y^2$ is called a Ricatti equation. Find the general solution of the Ricatti equation $y' = 1 + t^2 - 2ty + y^2$.

4. Determine the general solution of $18y''' + 21y'' + 14y' + 4y = 2e^t$.

5. Find the specific solution to the differential equation in Question 4 with initial conditions $y(0) = 0$, $y'(0) = 0$, $y''(0) = 2$.

6. Find the series solution (through t^5 terms) for the differential equation $(t^3 - 3)y'' + (2t - \cos t)y = 3$, $y(0) = 1$, $y'(0) = 2$.

7. Determine $\mathcal{L}[(t + 1)^2 \sin(at)]$.

8. Determine $\mathcal{L}^{-1}$ of your answer to the previous question, and compare to $(t + 1)^2 \sin(at)$.

9. Determine $\mathcal{L}^{-1}[1/((s + 1)^3(s^2 + 9))]$.

10. (a) Find the general solution of the linear system of differential equations

$$\mathbf{x}' = \begin{bmatrix} -8 & -10 & 7 & -9 \\ 0 & 2 & 0 & 0 \\ -9 & -9 & 8 & -9 \\ -1 & 1 & -1 & 2 \end{bmatrix} \mathbf{x} + \begin{bmatrix} e^t \\ 0 \\ \cos(3t) \\ 0 \end{bmatrix} .$$

(b) Find the specific solution of the system in 10(a) with the initial conditions

$$\mathbf{x}(0) = \begin{bmatrix} 2 \\ 0 \\ -5 \\ 3 \end{bmatrix} .$$

INSTRUCTIONS

Your assignment should be turned in from the computer print-out, stapled, and should be at most 2-3 pages long. It's possible that you need to bring your own paper for the printer (this is the case in MS 571). Make sure your print-out has your name, tutorial section, and ID number on it. One can type this in (or any other text) by clicking the Maple Menu at top on "T", typing in whatever is needed, and then "Enter". To go back to the Math Mode click the Menu on Σ, then on >.

Remember that each command ends with a ; followed by "Return", these will not be indicated below after 1(a). The specific commands for each problem follow. A couple of useful hints are first given. The basic arithmetic operations in Maple are $+$ $-$ $*$ $/$ $\wedge$. Be very careful about parentheses, there must always be as many left parentheses as right parentheses. The command % is a short-hand for the previous line's output. The exponential function e^x is typed $\exp(x)$. For some of the questions it's convenient to first define a function or a DE, this will be done first in Question 3.

1. (a)>evalf(exp(1),100); Enter   >evalf(Pi,100); Enter

   (b) just type in or write your answer here, but think carefully first - otherwise the chances are 99% you will get the wrong answer!

2. for the exact answer use $\text{int}(1/(1+x^6), x = 0..4)$
   for the numerical answer use   evalf(%)

3. First define "f" as follows $f := (x * \cos(x) - x)/(exp(\sin(x)) - exp(x))$
   the screen should now display f, so you can check your function.
   For the answer now use   limit(f,x=0) .

4. first define the DE $\text{de1}:=t*\cos(x(t))*\text{diff}(x(t),t)+(t+2)*\sin(x(t)) = 0$
   now   dsolve(de1,x(t))

5. like the previous question, first define   de2:= the DE (note x'' is written $\text{diff}(x(t),t,t)$,   similarly for x''')

6. use   dsolve($\{de2, x(0) = 0, D(x)(0) = 0, D(D(x))(0) = 2\}, x(t)$)

7. as before first label the DE   de3   and type it in
   now use   dsolve(de3,IC,$x(t)$,type=series)   (note here IC means enter the initial conditions as in the previous question)

   Before starting the Laplace transform, a general command is needed, type with(inttrans)

8. u:=$(t + 1)^2 * \sin(at)$
   laplace(u,t,s)

9. v:= the function of s found in your answer to Question 8
   invlaplace(v,s,t)
   add a few words of explanation as asked in Question 9

   Before doing Question 10 it will be necessary to undefine "v", since Maple will otherwise remember the "v" from Question 9. Use $v:='v'$,   also to undefine u use $u :=' u'$

10. now define v to be the function of s given in Question 10, and proceed as in Question 9.

To print just go to File and click on Print, similarly to exit go to File and click on Exit. It will ask you if you wish to save your work, generally the answer is No unless you wish to continue the session later.