AMAT 307 L03 ANNOUNCEMENT-QUIZ 3

This announcement gives some further explanation of the mixing question in Quiz 3, which asks for the long term behaviour of the solution. We will do the same question as Problem 5, Section 2.3, with different numbers.

Let's suppose the volume is 135(l), the inflow and outflow rate is 6 (l/min), the inflow concentration is $\frac{1}{2} + \frac{1}{4}\sin t$, and the initial amount of salt is 24(gm). The question is to find the long term solution. The steps will be outlined here - not all steps are given but it should not be hard to fill in the missing steps.

This leads to the IVP

$$Q' + \frac{2}{45}Q = 3 + \frac{3}{2}\sin t, \qquad Q(0) = 24.$$

This is a linear first order DE, so standard procedures give the general solution

$$Q = \frac{135}{2} + \left(\frac{3}{2}\right)\frac{(\frac{2}{45})\sin t - \cos t}{(\frac{2}{45})^2 + 1^2} + Ce^{-\frac{2t}{45}} .$$

Along the way the following indefinite integral, found in all tables of integrals, will be useful:

$$\int e^{ax}\sin(bx)dx = \frac{ae^{ax}\sin(bx) - be^{ax}\cos(bx)}{a^2 + b^2} .$$

Going back to the general solution, we see that as $t \to \infty$ the last term approaches zero, so as far as the long term behaviour the value of C (which the IC determines) makes no difference. The long term solution is thus

$$Q \to \frac{135}{2} + \left(\frac{3}{2}\right)\frac{(\frac{2}{45})\sin t - \cos t}{(\frac{2}{45})^2 + 1^2} .$$

Now, to answer the question on Quiz 3, one first needs the value about which the solution oscillates, this is clearly 135/2. The amplitude of the oscillation is less obvious because of the mixed $\sin t$, $\cos t$ term. However the $\cos t$ term clearly dominates, and the denominator is very close to 1, so forgetting the $\sin t$ term completely and taking the denominator equal to 1, we see that the amplitude of the oscillation is very close to 3/2. The answer should be given to 2 decimal places, and a more detailed analysis of the answer will show that as long as the ratio of input flow to volume (in this case $6/135 = 2/45 = .04444$) is $< 1/15 = .06666$, the approximation made will be good to 3 decimal places or better - and hence should be accurate enough for solving this question.

Hope this helps, if further questions email me or stop by Monday during office hrs (12:30-14:00). - PZ