AMAT 309 L01

Winter 2004
MIDTERM 50 Minutes

NAME: ______________________ ID: ________

1. Let $\mathbf{u}, \mathbf{v}, \mathbf{w}$, $\mathbf{r} = \mathbf{r}(t)$ be vectors in $\mathbb{R}^3$. For each of the following, answer with one of the words "vector, scalar, meaningless." [10]

   (a) $2\mathbf{u} - (\mathbf{v} \times \mathbf{w})$ ____________

   (b) $2\mathbf{u} - (\mathbf{v} \bullet \mathbf{w})$ ____________

   (c) $2\mathbf{u} \bullet (\mathbf{v} \times \mathbf{w})$ ____________

   (d) $(\mathbf{u} \bullet \mathbf{w}) \cdot (\mathbf{u} \times \mathbf{w})$ ____________

   (e) $\mathbf{r} \times (\mathbf{r}' \bullet \mathbf{r}'')$ ____________

2. For each of the following answer True or False. In (a), (b), $\mathbf{v}, \mathbf{w}$ denote any vectors in $\mathbb{R}^n$. [20]

   (a) $\| c \cdot \mathbf{v} \| = c \cdot \| \mathbf{v} \|$. ______

   (b) $\| \mathbf{v} + \mathbf{w} \| = \| \mathbf{v} \| + \| \mathbf{w} \|$. ______

   (c) At a point where $\kappa = 0$, $\mathbf{a} \times \mathbf{T} = \mathbf{0}$. ______

   (d) At a point where $\kappa = 0$, $\mathbf{a} \times \mathbf{N} = \mathbf{0}$. ______

   (e) If $a_T = 3$ and $a_N = 4$, then $a = 5$. ______

   (f) If $\mathbf{r}'''(t) = \mathbf{0}$, then $\tau = 0$. ______

   (g) For $z = f(x, y)$, ∇f will be orthogonal to the level curves.

   ______

   (h) If

   $$\lim_{(x,y)\to(a,b)} f(x, y) = L$$

   for every straight line path approaching (a, b), then this limit exists and equals L. ______

   (i) If the level curves of $z = f(x, y)$ are all circles, then the corresponding surface is a surface of revolution. ______

   (j) For any surface of revolution, the level curves are circles. ______

3. Let $z = f(w, x, y)$ and $w = s^2 - t^2$, $x = s + t$, $y = 3s - t$.
Determine $z_s(-3, 3, 1)$, given
$f_1(-3, 3, 1) = 1$, $f_2(-3, 3, 1) = 5$, $f_3(-3, 3, 1) = -1$, and
$f_1(0, 1, 2) = 3$, $f_2(0, 1, 2) = 4$, $f_3(0, 1, 2) = -2$. [20]

4. Determine, if it exists, [10]

$$\lim_{(x,y)\to(1,0)} \frac{x^2y - 2xy + y}{x^2 + 9y^2 - 2x + 1} \ .$$

5. (a) Identify and make a rough sketch of the surface $x^2/4 + y^2 = z^2/4 + 1$. [10]

(b) Find the equation of the tangent plane to this surface at $P = (-2, 1, 2)$. [10]

6. (a) Determine κ and τ for the curve $\mathbf{r}(t) = \langle t^3, -t^2, t^4 \rangle$. [15]

(b) Determine κ, τ for this curve at the point $P = (-1, -1, 1)$. [5]

SOLUTIONS

Question 1: vector, meaningless,scalar,vector,meaningless

Question 2: False,False,True,False,True,True,True,False,False,False

Question 3: First note that $w = -3$, $x = 3$, $y = 1$ implies $s = 1$, $t = 2$.

$$z_s = \frac{\partial z}{\partial s} = \frac{\partial z}{\partial w}\frac{\partial w}{\partial s} + \frac{\partial z}{\partial x}\frac{\partial x}{\partial s} + \frac{\partial z}{\partial y}\frac{\partial y}{\partial s}$$

$$z_s = \frac{\partial z}{\partial s} \cdot 2s + \frac{\partial z}{\partial x} \cdot 1 + \frac{\partial z}{\partial y} \cdot 3$$

Now evaluating we get $z_s = 1 \cdot 2 + 5 \cdot 1 + (-1) \cdot 3 = 4.$

Question 4: First use a little algebra to write in a little simpler form:

$$\lim_{(x,y)\to(1,0)} \frac{y(x-1)^2}{(x-1)^2 + 9y^2} \ .$$

Now make the substitution $x - 1 = r\cos\theta,\ 3y = r\sin\theta$. It reduces to

$$\frac{1}{3}\lim_{r\to 0}(r\sin\theta\cos^2\theta) \ = \ 0 \ .$$

Question 5: (a) It's a hyperboloid of 1 sheet. The sketch is best made by first drawing the "waist" of the hyperboloid, which is the ellipse $x^2/4 + y^2 = 1$ in the xy-plane, then roughly filling the rest.

(b) $x - 2y + z + 2 = 0$

Question 6: (a) Find first $\mathbf{v}, \mathbf{a}, \mathbf{a}'$, and then v, $\mathbf{v} \times \mathbf{a}$, $\| \ \mathbf{v} \times \mathbf{a} \ \|$. Then the standard formulas give

$$\kappa = \frac{2\sqrt{36t^4 + 64t^2 + 9}}{|t|(16t^4 + 9t^2 + 4)^{3/2}}, \qquad \tau = \frac{12}{t(36t^4 + 64t^2 + 9)} \ .$$

(b) Substituting $t = -1$ gives $\kappa = 2\sqrt{109}/29^{3/2}$, $\tau = -12/109$.