

# BONUS QUESTIONS

For credit on a bonus question turn in a written sheet with your name, course number, ID, and the solution - including your explanation.

1. Determine the next number in the sequence:

10 11 12 13 14 15 16 17 20 22 24 31 100 121 \_\_\_\_\_ .

2. A young lady named Eva is invited to a barbecue at a friend's house. When she arrives her friend greets her and she remarks "What a cute bunch of children playing in your garden." Her friend says "Yes, they actually belong to four families. My family is the largest, my brother's smaller, my sister's smaller yet, and my cousin's smallest of all. They wanted to play baseball but there were not enough to make up two teams, so they are playing a few other games. Curiously, the product of the numbers of children in the four families equals my house number." Eva then says "I happen to be a big fan of Sherlock Holmes, so since I know your house number let me see if I can figure out how many children are in each family." She does some calculations and finally says "Hmm, I'm getting close but could you kindly answer one more question: does your cousin's family consist of just a single child?" Her host answers this question, and then Eva tells just how many children are in each family.

Question: How many children are in each family?

3. Solve the anagram  $SEND + MORE = MONEY$  . Here each letter stands for a distinct digit, and one has to fill in the letter so that it makes sense as an addition problem. Also note that the first letters of the numbers, S and M, cannot equal 0. There is a unique solution, and to receive the bonus you should not only find the solution but explain (prove) that it is the only possible solution.
4. You are given 12 coins, which look identical except one of them is counterfeit. You are also given a balance (not a scale, just a balance).
  - (a) Warm-up question : Given that the counterfeit coin is heavier, find it using the balance just 3 times.
  - (b) Bonus question : Find the counterfeit coin using the balance just 3 times, knowing in advance only that it has a different weight from the others, but not knowing if it is lighter or heavier.
5. Find the four-dimensional volume of the 3-sphere of radius  $a$  :  $x_1^2 + x_2^2 + x_3^2 + x_4^2 = a^2$  . One suggested method (not necessarily the simplest) is to use 4-dimensional "spherical" coordinates, given by
$$x_1 = \rho \cos \theta \sin \phi \sin \psi$$

$$x_2 = \rho \sin \theta \sin \phi \sin \psi$$

$$x_3 = \rho \cos \phi \sin \psi$$

$$x_4 = \rho \cos \psi ,$$

$$0 \leq \rho \leq a, \quad 0 \leq \theta \leq 2\pi, \quad 0 \leq \phi, \psi \leq \pi .$$

You should show that the Jacobian reduces to  $\rho^3 \sin \phi \sin^2 \psi$ , then work out the appropriate quadruple integral.