1. Consider the *cissoid of Diocles*, which is the parametrized curve given by
$$c(t) = \left(\frac{2at^2}{1+t^2}, \frac{2at^3}{1+t^2} \right)$$
Sketch the curve if $a = 1/2$.

2. Show that the tangent lines to the parametrized curve $c(t) = (3t, 3t^2, 2t^3)$ make a constant angle with the line $y = 0$, $z = x$.

3. If $c(t)$ is a curve such that $\ddot{c}(t) = 0$ for all t, what can you say about c?

4. If $c(t) = (2t, t^2, t^3/3)$,
   (a) find the velocity, speed and acceleration for arbitrary t, and at $t = 1$.
   (b) find the arc-length function $s = s(t)$ (based at $t = 0$), and determine the arc-length of c from $t = -1$ to $t = 1$.

5. Show that the curve $c(t) = (t \cos t, t \sin t, t)$ lies on a cone in R^3. Find the velocity, speed and acceleration at the vertex of the cone.

6. Find an arc-length parametrization of the curve $c(t) = (\cosh t, \sinh t, t)$.

7. Let F be a fixed frame and f a moving frame in $\mathbb{R}^3$. If a vector $v = OX$ in the fixed frame, the tip has coordinates X in the fixed frame and x in the moving frame, and they are related by
$$X = Ax + b.$$
   (a) Prove the Coriolis theorem, which says that the velocities are related by
$$\dot{X} = A(\dot{x} + \omega \times x + A^{-1}\dot{b}).$$
   (b) Differentiate once more to show that the accelerations are related by
$$\ddot{X} = A(\ddot{x} + 2\omega \times \dot{x} + \omega \times (\omega \times x) + \dot{\omega} \times x + A^{-1}\ddot{b}).$$

Here ω is the vector corresponding to the antisymmetric matrix $\hat{\omega}$ which is defined by the equation

$$\dot{A} = A\hat{\omega}.$$

In the mechanics literature, ω is called the angular velocity vector in the body, and the vector Ω determined by $\dot{A} = \hat{\Omega}A$ is the called the angular velocity in space.