1. Let a, b and c be three vectors in $\mathbb{R}^3$. Show that *Jacobi's identity* for the cross product holds:

$$a \times (b \times c) + b \times (c \times a) + c \times (a \times b) = 0.$$

Remark: This is completely trivial to prove if you first establish the lemma that $[\hat{a}, \hat{b}] = \widehat{a \times b}$. Remember that $\hat{a}$ is the antisymmetric matrix

$$\hat{a}_{jk} = \sum_{l=1}^{3} \epsilon_{jkl} a_l,$$

where ϵ_{jkl} is Levi-Civita's permutation symbol.

2. Consider the curve $c(s) = (0.8 \cos s, 1 - \sin s, -.6 \cos s)$.

   (a) Show the curve is parametrized by arclength.

   (b) Compute the Frenet frame of the curve.

   (c) Compute the derivative of the Frenet frame with respect to the arc parameter s, and use Frenet's equations to identify the curvature κ and the torsion τ.

   (d) Show that the curve is a circle, and identify its center and radius.

3. Consider the curve

$$\beta(t) = \left(\frac{(1+t)^{3/2}}{3}, \frac{(1-t)^{3/2}}{3}, \frac{t}{\sqrt{2}} \right)$$

defined on the interval $-1 < t < 1$. Find an arclength parametrization of β, and compute the Frenet frame.

4. If D is the vector field $\tau T + \kappa B$ on a curve $c(s)$ parametrized by arclength, show that the Frenet formulas become

$$\begin{aligned} T' &= D \times T \\ N' &= D \times N \\ B' &= D \times B \end{aligned}$$

D is called the *Darboux vector*, after the French mathematician Gaston Darboux. His textbook on several variable calculus is still worth reading. You should also compare the matrix $\hat{D}$ with the matrix form of Frenet's equations that I gave in class.