AMAT 309 Assignment 1 Winter Term, 2006

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We encourage students to work together and to share ideas, and to talk to your distinguished lecturers about difficulties. However, your write-up should be your own, and it should be characterized by neatness and precise, concise discussion. Pictures from Maple or other graphing devices or software are perfectly acceptable. However, keep in mind that you will not have these aids on exams, so simple graphing (see problems 1 and 2) is a skill you need to develop.

1. Sketch the level curves of the surface $z = y\,(x^2 + 1)$.

2. Sketch the graph of $y = \ln(\cos(x))$, $-\frac{\pi}{2} < x < \frac{\pi}{2}$.

   What is the equation of the osculating circle at $(0, 0)$?

   What is $\kappa(x)$?

3. The formula

$$\left(\frac{\partial z}{\partial x}\right)\left(\frac{\partial x}{\partial y}\right)\left(\frac{\partial y}{\partial z}\right) = -1$$

   is used in thermodynamics. Interpret this equation, and prove it. (Hint: Think implicit functions)

4. Find the point(s) of maximum curvature on the graph of $y = \sinh x$.

5. (a) Show that the surfaces $z = x^2 y$ and $y = \frac{1}{4}x^2 + \frac{3}{4}$ intersect orthogonally at $(1, 1, 1)$.

   (b) Show that the surfaces $z = f(x, y) = -x^2 - y^2$ and $z = g(x, y) = \frac{1}{4}\ln(xy)$ intersect orthogonally along their entire curve of intersection.

6. (Problem 24, p. 742). Given a function $u(r,\theta)$ in polar coordinates, show that the Laplacian operator $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}$ becomes

$$\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r} + \frac{1}{r^2}\frac{\partial^2 u}{\partial \theta^2}.$$

7. Given the system of equations $x = u^2 - uv,\ y = 3uv + 2v^2$:

   (a) Show that this system can be solved for u and v as functions of x and y in a neighborhood of the point $(u,v,x,y) = (-1,2,3,2)$.

   (b) Find $\frac{\partial u}{\partial x}$ and $\frac{\partial v}{\partial x}$ at this point.

   (c) Use the linear approximation to estimate u and v for $(x,y) = (2.9, 2.02)$.

8. (This is Challenging Problem #1, p 781 in Adams)

   (a) If the graph of a function $z = f(x,y)$ that is differentiable at (a,b) contains part of a straight line through (a,b), show that the line lies in the tangent plane to $z = f(x,y)$ at (a,b).

   (b) If $g(t)$ is a differentiable function of t, describe the surface $z = y\,g(x/y)$ and show that all its tangent planes pass through the origin.