AMAT 309 Assignment 2   Winter Term, 2006

M. Hamilton, J. Macki

We encourage students to work together and to share ideas, and to talk to your instructors about difficulties. However, your write-up should be your own. Also, we encourage you to pay attention to presentation: solutions should be clear, logical, and well-presented. Pictures from Maple or other graphing devices or software are perfectly acceptable; however, keep in mind that you will not have these aids on exams, so simple graphing is a skill you need to develop.

1. (8) Find and classify the extrema of $h(x, y) = \sin(x)\sin(y)\sin(x + y)$ on the square$[0, \pi] \times [0, \pi]$. (Keep in mind there is a boundary to check out).

2. (8) Find those points on the curve of intersection of the surfaces $x^2 - xy + y^2 - z^2 = 1$ and $x^2 + y^2 = 1$ which are nearest the origin. (Hint: what was Lagrange's first name?)

3. (8) Find the volume of the finite solid bounded by the surfaces

$$az = x^2 + y^2, \ \ x^2 + y^2 + z^2 = 2a^2.$$

4. (8) Set up the correct limits for both iterated integrals for $\iint f(x, y)\, dA$ over D if D is:

   (a) The parallelogram with sides

   $$x = 3, \ x = 5, \ 3x - 2y + 4 = 0, \ 3x - 2y + 1 + 0.$$

   (b) The triangle with sides $y = 0, \ y = x, \ y = 4 - x$.

   (c) The finite domain cut out by the curves $y = x^2, \ y = 4 - x^2$.

   (d) The region bounded by the curve $\frac{(x-2)^2}{4} + \frac{(y-3)^2}{9} = 1$.

5. (8) Transform the following to polar coordinates, and evaluate:

   (a)
$$\int_0^2 dx \int_0^{\sqrt{4-x^2}} \ln\left(1+x^2+y^2\right)\, dy,$$

   (b)
$$\iint \arctan\left(\frac{y}{x}\right)\, dx, dy$$

   over the region defined by
$$1 \le x^2+y^2 \le 9, \quad \frac{x}{\sqrt{3}} \le y \le x\sqrt{3}.$$

6. (8) Change to cylindrical or spherical coordinates and evaluate:

   (a)
$$\int_0^2 dx \int_0^{\sqrt{2x-x^2}} dy \int_0^a z\sqrt{x^2+y^2}\, dz,$$

   (b) The volume of the solid that lies above the cone $\phi = \frac{\pi}{3}$ and below the sphere with equation $\rho = 4\cos(\phi)$

7. (4) Prove that the function
$$y(x) = \int_0^\infty \frac{e^{-xz}}{1+z^2}\, dz$$
   satisfies the differential equation $y''(x) + y = \frac{1}{x}$.

8. (4) Given a region $D \subset \mathbb{R}^2$ in the plane and a function of two variables $f(x,y)$, let R be the region in space above D and below the graph of f. Then we have two expressions for the volume of R, namely
$$\iint_D f(x,y)\, dA \qquad \text{and} \qquad \iiint_R dV$$
   Show that these two expressions are equal.

9. (8) Let $f(x,y) = 3x^4 - 4x^2y + y^2$

   (a) Show that on each line $y = mx$, the function has a minimum at 0.

   (b) Show that $(0,0)$ is not a minimum of f.

   (c) Make a sketch (or use Maple) showing those points (x,y) where $f(x,y) > 0$ and $f(x,y) < 0$.

   Hint: If you have trouble with part (b), perhaps try part (c) first.