AMAT 309 Quiz 1 Solutions Winter, 2006

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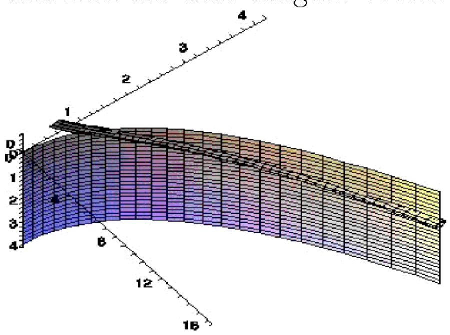
1. Simplify as much as possible:

$$[\mathbf{u}(x)\cdot(\mathbf{u}'(x)\times\mathbf{u}''(x))]' = \mathbf{u}(x)\cdot(\mathbf{u}'(x)\times\mathbf{u}'''(x))\,.$$

2. Sketch the curve

$$\mathbf{r}(t) = t\mathbf{i} + t^2\mathbf{j} + \ln(t)\mathbf{k}, \quad 1 \le t \le e,$$

and find the unit tangent vector $\hat{\mathbf{T}}(t)$ at the point (1,1,0).



The curve lies on the surface $y = x^2$, so we have graphed the two together using Maple, to help with visualization. Now

$$r'(t) = \mathbf{i} + 2t\mathbf{j} + \frac{1}{t}\mathbf{k},$$

which at $(1,1,0)$, *i.e.*, $t = 1$, gives

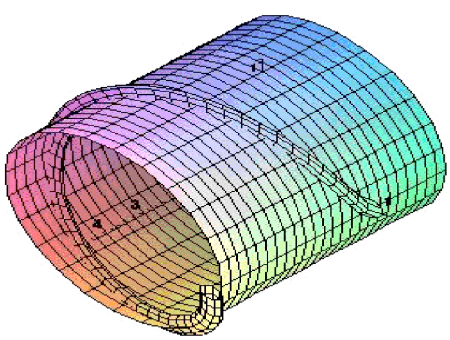
$$r'(1) = \mathbf{i} + 2\mathbf{j} + \mathbf{k},$$

so the unit tangent vector is

$$\hat{\mathbf{T}}(1) = \frac{1}{\sqrt{6}}(\mathbf{i} + 2\mathbf{j} + \mathbf{k})\,.$$

3. Sketch, and find the arclength of, the curve

$$\mathbf{r}(t) = 2t^{\frac{2}{3}}\mathbf{i} + \cos(2t)\mathbf{j} + \sin(2t)\mathbf{k},$$



$0 \le t \le \pi$.

The curve lies on the cylinder $y^2 + z^2 = 1$ so we have graphed the two together for visualization. Now

$$\mathbf{r}'(t) = \frac{4}{3}t^{-\frac{1}{3}}\mathbf{i} - 2\sin(2t)\mathbf{j} + 2\cos(2t)\mathbf{k},$$

so the arclength in question is given by

$$\int_0^\pi |\mathbf{r}'(t)|\, dt = \int_0^\pi \left[\frac{16}{9} t^{-\frac{2}{3}} + 4\right]^{\frac{1}{2}} \, dt =$$

$$= \frac{2}{3} \int_0^\pi \left[4 + 9t^{\frac{2}{3}}\right]^{\frac{1}{2}} t^{-\frac{1}{3}} \, dt =$$

$$= \frac{2}{27} \left[4 + 9t^{\frac{2}{3}}\right]^{\frac{3}{2}} |_0^\pi == \frac{2}{27} \left[4 + 9\pi^{\frac{2}{3}}\right]^{\frac{3}{2}} - \frac{2}{27} 8.$$