1. (a) Find a differential equation for the family of tangents to $x^2+y^2 = 1$. (Hint: Let $(\cos c, \sin c)$ be a point on the circle).
   (b) Show that a general solution of the differential equation in (a) is defined by the family of tangent lines.
   (c) Show that $x^2 + y^2 = 1$ is a solution of the differential equation of (a). What kind of solution is it?

2. Solve the following
   (a) $\sin y \; dx + (x \cos y - y)\, dy = 0$.
   (b) $(2y + 3x)dx + xdy = 0$.
   (c) $y' = \frac{y}{x} + \frac{x}{y}$.
   (d) $xyy' + y^2 = \sin x$. (Try $y^2 = u$.)

3. Show that the differential equation $y' = P(x)F(y) + Q(x)G(y)$ can be reduced to a linear differential equation by the transformations
$$u = \frac{F(y)}{G(y)} \quad \text{or} \quad u = \frac{G(y)}{F(y)}$$
according as
$$\frac{FG' - GF'}{G} \quad \text{or} \quad \frac{FG' - GF'}{F}$$
is a constant. Use this result to obtain the general solution of Bernoulli's equation
$$y' = P(x)y + Q(x)y^n.$$

4. A 10 lb object is dropped from a very high cliff. The law of resistance in the fps system is given by $0.001v^2$, where v is the instantaneous velocity. Determine
   (a) The velocity as a function of distance,
   (b) The velocity as a function of time,
   (c) The limiting velocity.

5. Find the value of the constant a so that the families $y^3 = c_1 x$ and $x^2 + ay^2 = c_2$ are orthogonal.