Applied Mathematics 311 Assignment #2

due date: November 7, 2005

1. Solve the following

   (a) $(D^2 - D + 1)y = x^3 - 3x^2 + 1$.

   (b) $y'' + y = x \sin x + \cos x$.

   (c) $(D^3 - 5D^2 - 2D + 24)y = x^2 e^{3x}$.

   (d) $x^2 y'' + xy' + 4y = 1$. Hint: try letting $x = e^z$ and let z be the new independent variable.

2. For what value of the constant p will $y = x^p$ be a solution of

$$x^2 y'' + 3xy' + y = 0?$$

   Write the general solution.

3. A weight W is suspended from a vertical spring and produces a stretch of magnitude a. When the weight is in equilibrium it is acted upon by a force which imparts to it a velocity v_0 downwards. Show that the weight travels a distance $v_0\sqrt{a/g}$ for a time $(\pi/2)\sqrt{a/g}$ before it starts to return.

4. This exercise extends the tutorial discussion.

   (a) The differential equation for the motion of a mass m suspended from a vertical spring of constant k, if damping is proportional to the instantaneous velocity is taken into account, is

$$m\ddot{x} + \beta\dot{x} + kx = 0,$$

   where the dots indicate differentiation with respect to time t. Show that damped oscillations will take place provided that the damping constant is small enough so that $\beta < 2\sqrt{km}$ and that x is given by

$$x(t) = Ce^{-\beta t/2m} \sin(\omega t + \phi)$$

   where $\omega = \sqrt{k/m - \beta^2/4m^2}$ and C and ϕ represent two arbitrary constants.

(b) i. Show that the times at which $x = Ce^{-\beta t/2m}\sin(\omega t + \phi)$ is a (local) maximum or minimum are given by $t_1, t_2, \ldots,$ where

$$t_n = \frac{1}{\omega}\left[\arctan\frac{\omega}{\alpha} + (n-1)\pi - \phi\right]$$

Hence show that the quasi period is $2\pi/\omega$.

ii. Show that the quasi period for the motion described in the previous section is greater than the natural period.

(c) By using the result of the previous section show that the successive maximum distances from the equilibrium position are given by

$$x_n = Ce^{-\beta t_n/2m}\sqrt{1-\beta^2/4mk}$$

where the t_n are given in the last section. Hence show that

$$\frac{x_{n+1}}{x_n} = e^{-\beta\pi/2m\omega}.$$

That is to say, the successive swings decrease in geometric progression. In engineeering, the quantity $\beta\pi/2m\omega$ is called the *logarithmic decrement.*