Applied Mathematics 311 | Assignment #3

due date: November 28, 2005

1. Use Picard's method to obtain a solution to $y' = x^2 - y$; $y(0) = 0$. Find at least the fourth approximation to the solution.

2. Find Frobenius-type solutions for the following about the point $x = 0$.

   (a) $x^2y'' + xy' + (x^2 - \frac{1}{4})y = 0$.

   (b) $xy'' + y' + y = 0$.

3. Show that the roots of the indicial equation may be found by letting $y = x^c$ in the given differential equation and determining c so that the coefficient of the lowest power of x is zero. Illustrate this on one of the equations in problem 2.

4. Show that if x is replaced by $x = \lambda z$, where λ is a constant, then Bessel's differential equation becomes

$$z^2\frac{d^2y}{dz^2} + z\frac{dy}{dz} + (\lambda^2z^2 - n^2)y = 0.$$

   Hence solve $4x^2y'' + 4xy' + +(2x^2 - 1)y = 0$.

5. (Extra for experts) In his efforts to solve Kepler's equation, Bessel arrived at the eponymous function of x defined by the integral

$$y(x) = \frac{1}{2\pi}\int_0^{2\pi} \cos(n\theta - x\sin\theta)\, d\theta.$$

   Show that for $n = 0$ we have $y = J_0(x)$. One way to do this is to use the Maclaurin series for $\cos u$ where $u = x\sin\theta$ and then integrate term by term.