Make yourself a single $8\frac{1}{2} \times 11$ formula sheet, and see if you can do the following practice exam in 3 hours.

1. What is the condition for
$$f(x)\,dx + g(x)h(y)\,dy = 0$$
to be an exact equation?

2. Solve
   (a) $xy' + y = x^2, \qquad y(1) = 2.$
   (b) $x\,dy - y\,dx = x^2 y\,dy.$

3. Find the orthogonal trajectories of $x^p + cy^p = 1$, p a constant.

4. When light passes through a window, some of it is absorbed. Experimentally, the amount of light absorbed by a small thickness of glass is proportional to the thickness of the glass and to the amount of incident light. Show that if r percent of the light is absorbed by a thickness w, then the percentage of the light absorbed by a thickness nw is
$$100\left[1 - \left(1 - \frac{r}{100}\right)^n\right], \qquad 0 \le r \le 100$$

5. A point moves in the first quadrant of the xy plane so that the tangent to its path makes with the coordinate axes a triangle whose area is equal to the constant a^2. Find the path.

6. Check that x^2 is a solution of
$$x^2y'' - 2xy' + 2y = 0$$
Then reduce the order and hence find the general solution.

7. Find the inverse Laplace transform
$$\mathscr{L}^{-1}\left\{\frac{e^{-2s}}{s^3}\right\}.$$

8. Solve

$$tY'' - tY' + Y = 0, \quad Y(0) = 0, \quad Y'(0) = 1.$$

Hint: take the Laplace transform.

9. Show that

$$xy'' + (1-x)y' + 2y = 0, \qquad y(0) = 2$$

has a quadratic polynomial as a solution. You may find it beneficial to solve this with the Frobenius 'machine'.

10. Find a nontrivial solution about $x = 0$ of

$$xy'' + y = 0$$