In order to study sensibly for the midterm, it helps to have a general sense of what we have covered so far.

We first looked at first order ode, and the idea of exactness. We then looked at some cases that were made exact by a suitable integrating factor, and the method of separation of variables. An important special case was the first order linear differential equation. There were also a couple of special tricks such as equations with one variable missing and homogeneous equations.

As applications of these techniques, we looked at some problems in mechanics such as falling under the influence of gravity, orthogonal trajectories, mixing problems, and the problem of the hanging cable.

We then looked at linear differential equations, and found some techniques for finding the complementary solution, and the case of repeated roots. We then looked at finding the particular solution, with the method of undetermined coefficients and the method of variation of parameters. Once again, there are a couple of general techniques such as reduction of order, for finding lower order equations once we know particular solutions.

As an application of these ideas we rather exhaustively looked at the equation of forced vibrations, with the concomitant notions of underdamped, critically damped and resonance. We then observed that this analysis applies to some electric circuits.

Solving the following problems will show that you are in fine shape for the midterm. For the midterm you may bring in a single $8\frac{1}{2} \times 11$ sheet of paper with anything you wish hand written on it (both sides even.) I do not care if you hand copy your friend's cheat sheet, but you may not photocopy it. It is important to note for your peace of mind that since the midterm is only 50 minutes, the problems will be simpler than those on the assignments.

1. Solve $y' + y \tan x = 0$.

2. Show that $\cos x\, dy - (2y \sin x - 3)\, dx = 0$ is not exact, but becomes exact upon multiplying by the integrating factor $\cos x$. Hence, solve the equation.

3. Solve $y\, dx + +(x^3 y^2 + x^3)\, dy = 0$.

4. A particle moves along the x-axis acted upon only by an opposing force proportional to its instantaneous velocity. It starts at the origin with a velocity of 10m/s, which is reduced to 5m/s after moving 2.5m. Find its velocity when it is 4m from the origin.

5. Find the orthogonal trajectories of the family $y^3 = cx^2$.

6. A tank has 100 gal brine and 40lb of dissolved salt. Pure water enters at 2 gal/min and leaves at the same rate. When will the salt concentration be less than 0.01lb/gal?

7. Uranium disintegrates at a rate proportional to the amount present at any instant. If M_1 and M_2 grams of uranium are present at times T_1 and T_2 respectively, show that the half-life of uranium is

$$\frac{(T_2 - T_1)\ln 2}{\ln(M_1/M_2)}.$$

8. Find the equation of a curve passing through $(1, 1)$ having the property that the x-intercept of its tangent line equals the y-intercept of its normal line.

9. Solve

   (a) $(D^3 + 2D^2 - 5D - 6)y = 0$.

   (b) $y''' = y''$.

   (c) $(D^3 + D)y = x + \sin x + \cos x$.

   (d) $y'' + 4y = \csc 2x$.

10. A 3lb weight on a spring stretches it 6 in. Assuming a damping force numerically equal to βv, where v is the instantaneous velocity in feet per second and $\beta > 0$, for what value of β is the motion critically damped?

11. A capacitor of 0.001 farads is in series with an emf of 20 volts and an inductor of 0.4 henries. At $t = 0$, $Q = 0$, and $I = 0$.

    (a) Find the natural frequency and period of the electric oscillations.

    (b) Find the maximum charge and current.