Winter 2004

AMAT 411
Assignment 3

Due date: March 12. Only * problems will be graded.

1. Solve the system $y' = \begin{pmatrix} -1 & 1 & 1 \\ 0 & 0 & 1 \\ -3 & 2 & 0 \end{pmatrix} y$

* 2. Solve the system $y' = \begin{pmatrix} 0 & 1 & 0 & 1 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \end{pmatrix} y$

* 3.(a) Solve the inhomogeneous system

$$\begin{cases} x' = -2x - 5y + 1 \\ y'' = 2x + 5y + 2y', \end{cases}$$

[Careful: only 3 arb. constants]

satisfying $x(0) = \frac{2}{9}$, $y(0) = 0$, $y'(0) = -\frac{2}{9}$.

(b) Solve the initial value problem for

$$y' = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ -1 & 2 & 2 \end{pmatrix} y + \begin{pmatrix} t \\ 0 \\ t \end{pmatrix}, \quad y(0) = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}.$$

* 4. Solve

$$\begin{cases} x' = 4x + 3y - 2 - 4t \\ y' = -x + t \end{cases}$$

5. Reduction of Order (when one solution is known)

(a). If (x_1, y_1) is a solution of the system

$$x' = ax + by$$
$$y' = cx + dy,$$

show that the change of variable

$$\begin{cases} x = x_1 v \\ y = y_1 v + z \end{cases}$$

reduces the system to
$$\begin{cases} v' = \frac{b}{x_1} z, \\ z' = \left(d - \frac{y_1}{x_1} b\right) z. \end{cases}$$

* (b) Given that $(t, 1)$ is a solution of the system

$$x' = x + (1-t)y$$
$$y' = \frac{x}{t} - y \quad , \quad t > 0,$$

solve the system.