Name _____

1. Suppose that we start with the digital signal f_k and define a new digital signal by interleaving zeroes between its samples. That is,

$$g_k = \begin{cases} f_{k/2} & k \text{ even} \\ 0 & k \text{ odd} \end{cases}$$

Find the z-transform $\mathscr{G}(z)$ of g_k in terms of the z-transform $\mathscr{F}(z)$ of f_k. This operation of interleaving zeroes is used in oversampling.

2. Find the z-transform of the signal

$$f_k = \begin{cases} 1/k! & k \geq 0 \\ 0 & k < 0 \end{cases}$$

Do not forget that $0! = 1$.

3. Compute the z-transform of the following sequences

   (a) $x_t = e^{-at} u_t$.

   (b) $x_t = e^{-at} \sin(\omega t) u_t$.

4. Suppose that the transfer function of a digital filter is given by

$$\frac{(z-1)^2}{z^2 + (m_1 - m_2)z + (1 - m_1 - m_2)}.$$

Plot a graph specifying the region of the $m_1 - m_2$ plane in which the digital filter is stable.

5. Leonardo Fibonacci's classic sequence as a filter problem. Suppose for the sake of argument that a pair of rabbits is born, matures in one month, and produces a pair of baby rabbits a month after reaching maturity, and then another pair every month thereafter. Start with a single pair of rabbits at the beginning of month zero. At the beginning of month one this pair matures, but still there is only one pair of rabbits. By the beginning of month two, there is the original pair, plus one new baby pair. By month three there are three pairs. However, by month four there will be five pairs of rabbits, the three pair from the preceeding month plus two new baby pairs.

   Denote the number of rabbits at the beginning of month t by r_t.

   (a) Derive an expression for r_t in terms of r_{t-1} and r_{t-2}.

   (b) Interpret r_t as the output signal of an appropriate digital filter with appropriate initial conditions. Is this a feedforward or feedback filter?

   (c) Is the filter stable?

   (d) Find $\mathscr{R}(z)$, the z-transform of r_t.

   (e) Find the poles and the corresponding partial fraction expansion of $\mathscr{R}(z)$.

   (f) Find an explicit expression for r_t by taking the inverse z-transform of the partial fraction expansion.

   (g) How many pairs of rabbits will there be after one year?