Name _____

A single $8\frac{1}{2} \times 11$ formula sheet is allowed, but a calculator is not.

1. Let $f(z)$ be the complex function defined by

$$f(z) = \frac{1}{2+3z}.$$

   (a) This function has a Taylor series converging about the origin. Find this series (i.e. expand about $z = 0$) and state the radius of convergence $\rho$ of the series. [5]

   (b) This function has a Laurent series converging in $|z| > \rho$. Find this series (i.e. expand in powers of $1/z$.) [5]

2. (a) Find the location, order and residue of each pole of [5]

$$f(z) = \frac{e^{i2z}}{z^2+1}$$

   (b) Find the integral [5]

$$\int_{-\infty}^{\infty} \frac{\cos 2x}{x^2+1}\, dx$$

3. What frequency do we see if we sample a 531 Hz sinusoid at 300 Hz? [10]

4. Consider the digital filter defined by

$$y_t = \frac{1}{3}(x_t + x_{t-1} + x_{t-2}).$$

   Find the magnitude response of this filter, and explain why you would consider this filter more of a lowpass or highpass filter. [10]

5. Consider the function $f(x) = \sin^3 x$. Find its Fourier series. [10]

6. Let $x_t = \{\ldots, 0, 0, 1, 2, 3, 0, 0, \ldots\}$, with $x_0 = 1, x_1 = 2$ etc. Find the $z$-transform of the convolution $x * x$. [10]

7. Consider the filter with input $x_t = \delta_t$, and output $y_{-1} = 1/4$, $y_{-2} = 0$, and the other $y_t$ determined by [10]

$$4y_t + 4y_{t-1} + y_{t-2} = x_t$$

for $t \geq 0$. Is the output $y_t$ bounded for all $t$?

8. Suppose you are given 4096 sampled points $x_t$ of a continuous signal which is supposed to be of the form $x(t) = A\cos(\omega t + \phi) + \text{noise}$. Suppose also that the samples are taken exactly 1 second apart. Explain in as much detail as you can the procedure you would use to estimate the unknown constants $A, \omega, \phi$. [10]