NAME ______________________________

1. Write in the form $a + bi$:
$$\frac{5}{(1-i)(2-i)(3-i)}$$
2. Solve $z^2 - 2z + 2 = 0$.
3. Sketch the set of points in the complex plane $\mathbb{C}$ satisfying
$$|z-1| = |z+i|.$$
4. If $z \neq 1$ show that
$$1 + z + z^2 + \cdots + z^n = \frac{1-z^{n+1}}{1-z}.$$
Then derive Lagrange's trigonometric identity
$$1 + \cos\theta + \cos 2\theta + \cdots + \cos n\theta = \frac{1}{2} + \frac{\sin(n+\frac{1}{2})\theta}{2\sin\frac{\theta}{2}}.$$
5. Show that if $f(z) = \mathrm{Re}(z)$ then $f'(z)$ does not exist anywhere.
6. Show that $f(z) = e^{-y}e^{ix}$ is an analytic function.
7. Find a function $v(x, y)$ so that if $u(x, y) = 2x(1-y)$, $f(z) = u + iv$ is an analytic function. Such a v is called a *harmonic conjugate* of u.
8. Show that $\sin(iz) = i\sinh z$ and $\cos(iz) = \cosh z$.
9. Let C be the boundary of the square with vertices $0, 1, 1+i, i$ traversed counterclockwise. Compute the contour integral
$$\int_C \pi e^{\pi\bar{z}}\,dz.$$