AMAT 425 Asst 3 Solutions

10.8 (a) Phase 1: pick x_2 col, x_{-5} row (note $M_1 = 0$ and x_{-5} will become nonbasic, hence ending Phase 1, only after this pivot step)

(b) Phase 1: M_1 row shows optimality but $M_1 \neq 0$, so the feasible set is empty

(c) M row shows optimality with optimal value $M = 10$ at $x_1 = x_4 = x_5 = 0$, $x_2 = 5$, $x_3 = 2$

(d) Pick x_2' col, x_4 row (actually col 1 can be treated as in (e))

(e) x_3 col has $\leq \underset{\sim}{0}$ above < 0, so the objective M is unbounded

11.1 Primal problem is: min $2x_A + 3x_B + 5x_C + 6x_D + 8x_E + 8x_F$ (where x_A = number of oz of feed A, etc.) subject to $x_A, \ldots, x_F \geq 0$ and

$$\begin{bmatrix} 20 & 30 & 40 & 40 & 45 & 30 \\ 50 & 30 & 20 & 25 & 50 & 20 \\ 4 & 9 & 11 & 10 & 9 & 10 \end{bmatrix} \begin{bmatrix} x_A \\ \vdots \\ x_F \end{bmatrix} \geq \begin{bmatrix} 70 \\ 100 \\ 20 \end{bmatrix} \Big\} \text{units of } \begin{cases} \text{protein} \\ \text{carbs} \\ \text{fat} \end{cases}$$

$\therefore$ Dual problem is: max $70y_p + 100y_c + 20y_f$

subject to $y_p, y_c, y_f \geq 0$ and

$$\begin{bmatrix} 20 & 50 & 4 \\ 30 & 30 & 9 \\ 40 & 20 & 11 \\ 40 & 25 & 10 \\ 45 & 50 & 9 \\ 30 & 20 & 10 \end{bmatrix} \begin{bmatrix} y_p \\ y_c \\ y_f \end{bmatrix} \leq \left.\begin{bmatrix} 2 \\ 3 \\ 5 \\ 6 \\ 8 \\ 8 \end{bmatrix}\right\} ¢/\text{oz}$$

Units: the terms $2x_A$ etc. in the primal objective are in units of $\frac{¢}{\text{oz}} \cdot \text{oz}$ so the primal (and hence dual) objective is in ¢.

Thus $70y_p$ is in units of ¢ and since 70 is in units of protein, y_p must be in units of ¢/protein unit. Hence y_p is the (shadow) price of protein. Similarly y_c, y_f are the prices of carbs, fat (i.e., what the pet store is prepared to pay the wholesaler(s) for these basic ingredients of feeds A,…, F).

11.2 P. objective: min $57x_1 + 27x_2 + 73x_3 = 3010$ at $\underset{\sim}{x} = (40, 0, 10)$
$= $ dual max

P. 3rd constraint: $2x_1 + 2x_2 + 4x_3 - s_3 = 100$
gives 3rd slack $s_3 = 20$ at $\underset{\sim}{x} = (40, 0, 10)$.

By complementary slackness, the 3rd dual variable $y_3 = 0$ and the dual slacks $t_1 = t_3 = 0$ since $x_1 = 40$, $x_3 = 10$.

The original dual tableau* is

$$\begin{array}{c|ccc|c} & y_1 & y_2 & y_3 & \\ \hline t_1 & 3 & 4 & 2 & 57 \\ t_2 & 2 & 1 & 2 & 27 \\ t_3 & 4 & 5 & 4 & 73 \\ \hline & -160 & -210 & -100 & 0 \end{array}$$

and inserting $y_3 = t_1 = t_3 = 0$ into the 1st and 3rd equations gives

$$3y_1 + 4y_2 = 57$$
$$4y_1 + 5y_2 = 73.$$

Solving these we get $y_1 = 7$, $y_2 = 9$ (and $y_3 = 0$).

11.4 Primal: $\max \underset{\sim}{c}^T \underset{\sim}{x}$: $A\underset{\sim}{x} \le \underset{\sim}{b}$ and $\underset{\sim}{x} \geq 0$.
$-A\underset{\sim}{x} \le -\underset{\sim}{b}$

Thus the dual is $\min [\underset{\sim}{b}^T - \underset{\sim}{b}^T]\underset{\sim}{y}$: $[A^T - A^T]\underset{\sim}{y} \geq \underset{\sim}{c}$, $\underset{\sim}{y} \geq 0$.

Let A be $m \times n$, so $\underset{\sim}{y} \in \mathbb{R}^{2m}$ and we can write
$\underset{\sim}{y} = \begin{bmatrix} \underset{\sim}{p} \\ \underset{\sim}{q} \end{bmatrix}$ where $\underset{\sim}{p}, \underset{\sim}{q} \geq \underset{\sim}{0} \in \mathbb{R}^m$. Now write $\underset{\sim}{z} = \underset{\sim}{p} - \underset{\sim}{q}$,
so $\underset{\sim}{z}$ is unrestricted in sign and the dual problem becomes

$$\min \underset{\sim}{b}^T \underset{\sim}{z} : A^T \underset{\sim}{z} \geq \underset{\sim}{c}.$$

* dual problem: $\max [160 \;\; 210 \;\; 100]\underset{\sim}{y}$: $\begin{bmatrix} 3 & 4 & 2 \\ 2 & 1 & 2 \\ 4 & 5 & 4 \end{bmatrix} \underset{\sim}{y} \le \begin{bmatrix} 57 \\ 27 \\ 73 \end{bmatrix}$, $\underset{\sim}{y} \geq \underset{\sim}{0}$