

University of Calgary Department of Mathematics and Statistics
 Amat 433 L01 Oct. 25, 2002 Midterm Test I. Duration: 50 minutes
 No Calculators Allowed

FALL 2002

The marks for each problem are shown in brackets. Total marks = 80.

1. Let $f(x) = x$ for $0 < x < \pi$.

- [19] (a) Find the Fourier Sine coefficients, b_n , of f and the Fourier Sine series of f .
 [3] (b) To what number does the Fourier sine series of f converge at $x=2$? At $x=-1$? At $x=\pi$?
 [5] (c) Find the value of $\sum_{n=1}^{\infty} b_n^2$.

[20] 2. Solve the heat equation

$U_{xx} = U_t$ for $x > 0, t > 0$, with initial condition $U(x, 0) = e^{-x}$ $x > 0$ and boundary conditions $U(0, t) = 0$ $t > 0$, $U(x, t)$ bounded. Express your answer in manifestly real form.

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- [25] 3. Masses m_1 and m_2 are connected by three springs with spring constants k_1, k_2, k_3 .
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- The diagram shows two horizontal masses, m_1 and m_2 , connected by three springs. The first spring, labeled k_1 , connects the left end of mass m_1 to the right end of mass m_2 . The second spring, labeled k_2 , connects the left end of mass m_2 to the right end of mass m_1 . The third spring, labeled k_3 , connects the right end of mass m_1 to the right end of mass m_2 . The masses are represented by rectangles, and the springs are represented by horizontal lines with small circles at the points of connection.

The equations of motion for the position coordinates x_1 and x_2 are

$$m_1 \ddot{x}_1 = -k_1 x_1 - k_2(x_1 - x_2)$$

$$m_2 \ddot{x}_2 = -k_3 x_2 - k_2(x_2 - x_1)$$

In the case $m_1 = m_2 = k_1 = k_3 = 1$ and $k_2 = 2$, find the normal mode vectors $\begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$ and the normal mode frequencies of oscillation.

- [8] 4. Give an example of a non-diagonal 3×3 matrix A which satisfies
- $$A(A-I)(A-2I) = 0 \cdot I$$

Information

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Math 2400 Final Exam Oct 25, 2012 Version 1, Page 1, Duration: 50 minutes

$$\int x \sin \alpha x \, dx = \frac{\sin \alpha x}{\alpha^2} - \frac{x \cos \alpha x}{\alpha}$$

$$\int x \cos \alpha x \, dx = \frac{\cos \alpha x}{\alpha^2} + \frac{x \sin \alpha x}{\alpha}$$

$$\int e^{\alpha x} \sin \beta x \, dx = e^{\alpha x} [\alpha \sin \beta x - \beta \cos \beta x] \frac{1}{\alpha^2 + \beta^2}$$

$$\int e^{\alpha x} \cos \beta x \, dx = e^{\alpha x} [\alpha \cos \beta x + \beta \sin \beta x] \frac{1}{\alpha^2 + \beta^2}$$

[20] 2. Solve the heat equation

$U_{xx} = U_t$ for $x > 0, t > 0$, with initial condition $U(x, 0) = e^{-x}$, $x > 0$ and boundary conditions $U(0, t) = 0$, $t > 0$, $U(x, t)$ bounded. Express your answer in manifestly real form.