DEPARTMENT OF MATHEMATICS AND STATISTICS
AMAT 433 MIDTERM TEST I OCT. 29, 2004

Total Marks = 80.
Work all problems. Marks are shown in brackets.

Duration=50 minutes.

[23] 1. Solve the p.d.e. $U_{xx}+U_{yy}=0$ on $-\infty<x<\infty$, $0<y<b$ subject to the conditions

$U(x,y)$ bounded, $U(x,0)=0$,

$$U_y(x,b)=\begin{cases}1 & -a<x<a\\ 0 & |x|>a.\end{cases}$$

Express your answer in manifestly real form.

2. Define $f(x)$ by

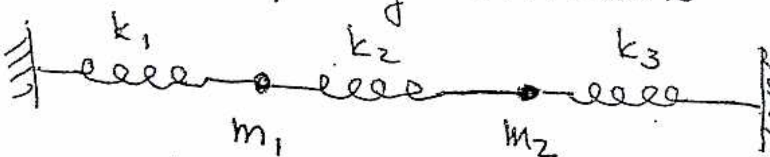
$$f(x)=\begin{cases}1 & -2<x<-1\\ 1 & 1<x<2\\ 0 & x<-2,\ -1<x<1,\ x>2.\end{cases}$$

[15] (a) Find the Fourier transform $g(w)$ of f. Express g in terms of w and trig functions.

[8] (b) Use the result of part (a) to find the value of I,

$$I\equiv\int_0^\infty \frac{\sin\lambda}{\lambda}\left(2\cos\lambda-1\right)\cos\lambda\, d\lambda.$$

[26] 3. Masses m_1 and m_2 are connected by three springs with spring constants k_1, k_2, k_3.



The equations of motion for the position coordinates x_1 and x_2 are:

$$m_1 \ddot{x}_1 = -k_1 x_1 - k_2(x_1 - x_2)$$

$$m_2 \ddot{x}_2 = -k_3 x_2 - k_2(x_2 - x_1).$$

In the case $m_1 = m_2 = k_3 = 1$, $k_1 = 4$, $k_2 = 2$ find the normal mode vectors $\begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$ and the normal mode frequencies of oscillation.

[8] 4. Suppose that A is a 4×4 matrix that has only one distinct eigenvalue. Suppose also that $\begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix}$ and $\begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}$ are eigenvectors of A and A has no eigenvectors linearly independent of these two. Prove that each of the vectors $\begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}$ and $\begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix}$ is a generalized eigenvector of A.

Information.

$$\int x \sin \alpha x \, dx = \frac{\sin \alpha x}{\alpha^2} - \frac{x \cos \alpha x}{\alpha}$$

$$\int x \cos \alpha x \, dx = \frac{\cos \alpha x}{\alpha^2} + \frac{x \sin \alpha x}{\alpha}$$

$$\int e^{\alpha x} \sin \beta x \, dx = e^{\alpha x} \left[\alpha \sin \beta x - \beta \cos \beta x \right] \frac{1}{\alpha^2 + \beta^2}$$

$$\int e^{\alpha x} \cos \beta x \, dx = e^{\alpha x} \left[\alpha \cos \beta x + \beta \sin \beta x \right] \frac{1}{\alpha^2 + \beta^2}$$

$$e^{iz} = \cos z + i \sin z$$

$$\cos z = \frac{e^{iz} + e^{-iz}}{2}$$

$$\sin z = \frac{e^{iz} - e^{-iz}}{2i}$$

$$\sin(2z) = 2 \sin z \cos z$$