UNIVERSITY OF CALGARY
DEPARTMENT OF MATHEMATICS AND STATISTICS
AMAT 433 L01 FINAL EXAMINATION DEC. 16, 2003

Total Marks = 120. Duration=2 hours.
Work all problems. Marks are shown in brackets.

[25] 1. Use complex analysis to find the (principal) value of the real integral

$$\int_{-\infty}^{\infty} \frac{\cos(2x)}{(x^4-1)}dx.$$

[13] 2. Find the value of

$$\int_C \frac{\sin(2z)dz}{(z+i)^3}$$

where the counterclockwise contour C is the circle of radius 2 centered at the origin of the complex plane.

[12] 3. Let

$$f(z) = z + 5 + \frac{z+e^z}{z-1}.$$

Find all a_n and all b_n in the Laurent series of $f(z)$ centered at $z_0 = 1$.

4. Let A be the complex matrix $A = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 1+i \\ 0 & 1-i & 0 \end{pmatrix}$.

[15] (a) Find the eigenvalues and eigenvectors of A.

[5] (b) Express A^{-1} as a linear combination of the matrices A^2, A, I.

[30] 5. Solve the potential equation $U_{xx} + U_{yy} = 0$ on $-\infty < x < \infty$, $0 < y < b$, subject to the conditions:

$$U(x,y) \text{ bounded}$$

$$U_y(x,0) = 0,$$

$$U(x,b) = \left\{ \begin{array}{ll} 1 & -a < x < a \\ 0 & |x| > a \end{array} \right\}.$$

Express your answer in manifestly real form.

[10] 6. (a) Let $H(t-a)$ be defined as

$$H(t-a) = \left\{ \begin{array}{ll} 1 & t > a \\ 0 & t < a \end{array} \right\}.$$

Find the Laplace transform of $H(t-a)$ and of $e^{\beta t}H(t)$.

[10] (b) The charge $q(t)$ on the capacitor in an LRC circuit driven by a voltage V is governed by

$$Lq'' + Rq' + \left(\frac{1}{C}\right)q = V.$$

Consider the special case $L = 0$, $R = 1$, $\left(\frac{1}{C}\right) = 2$ with initial condition $q(0) = 0$, and $V(t) = V_0 H(t-a)$. So V is a constant voltage V_0 turned on at time $t = a$. Use Laplace transform methods to find $q(t)$.

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INFORMATION:

$$\frac{1}{(x+\alpha)(x+\beta)} = \frac{1}{\beta-\alpha}\left[\frac{1}{x+\alpha} - \frac{1}{x+\beta}\right]$$