APPLIED MATHEMATICS 605 ASSIGNMENT #1

DUE DATE: OCTOBER 19, 2009

1. For $f(x) = 2x(1-x)$ find the limit $\lim_{n\to\infty} f^n(0.25)$.
2. Give an example of a one-to-one continuous onto map $f : X \to Y$, with X, Y topological spaces, that is not a homeomorphism.
3. Show that any perfect subset of the real line is uncountable.
4. Let $f : \mathbb{R} \to$ be continuous, $n > 3$, and $x_1, x_2, \ldots, x_n$ be such that $x_1 < x_2 < \cdots < x_n$. Show that if $f(x_k) = x_{k+1}$ for $1 \leq k \leq n-1$, $f(x_n) = x_1$, then f has points with all prime periods.
5. Draw the bifurcation diagram of $f_c(x) = x^3 + cx$.
6. Let D be an interval and $f : D \to D$ be a continuous function.
   (a) Suppose that for any nontrivial closed intervals I and J contained in D, we can find n so that $J \subset f^n(I)$. Prove that f is chaotic on D.
   (b) Is the converse of part (a) true?