

**FACULTY OF SCIENCE
THE UNIVERSITY OF CALGARY
DEPARTMENT OF MATHEMATICS AND STATISTICS
FINAL EXAMINATION
MATH 205 L01**

Date: Saturday, April 20, 2002

Time: 3:30-6:30 pm

Winter 2002

I.D. NUMBER	SURNAME	OTHER NAMES
	KEY	

STUDENT IDENTIFICATION

Each candidate must sign the Seating List confirming presence at the examination. All candidates for final examinations are required to place their University of Calgary student I.D. cards on their desks for the duration of the examination. (Students writing mid-term tests can also be asked to provide identity proof.) Students without an I.D. card who can produce an acceptable alternative I.D., e.g., one with a printed name and photograph, are allowed to write the examination.

A student without acceptable I.D. will be required to complete an Identification Form. The form indicates that there is no guarantee that the examination paper will be graded if any discrepancies in identification are discovered after verification with the student's file. A student who refuses to produce identification or who refuses to complete and sign the Identification Form is not permitted to write the examination.

EXAMINATION RULES

1. Students late in arriving will not normally be admitted after one-half hour of the examination time has passed.
2. No candidate will be permitted to leave the examination room until one-half hour has elapsed after the opening of the examination, nor during the last 15 minutes of the examination. All candidates remaining during the last 15 minutes of the examination period must remain at their desks until their papers have been collected by an invigilator.
3. All enquiries and requests must be addressed to supervisors only.
4. Candidates are strictly cautioned against:
 - (a) speaking to other candidates or communicating with them under any circumstances whatsoever;
 - (b) bringing into the examination room any textbook, notebook or memoranda not authorized by the examiner;
 - (c) making use of calculators and/or portable computing machines not authorized by the instructor;
 - (d) leaving answer papers exposed to view;
 - (e) attempting to read other students' examination papers.

The penalty for violation of these rules is suspension or expulsion or such other penalty as may be determined.

5. Candidates are requested to write on both sides of the page, unless the examiner has asked that the left half page be reserved for rough drafts or calculations.
6. Discarded matter is to be struck out and not removed by mutilation of the examination answer book.
7. Candidates are cautioned against writing in their answer books any matter extraneous to the actual answering of the question set.
8. The candidate is to write his/her name on each answer book as directed and is to number each book.
9. A candidate must report to a supervisor before leaving the examination room.
10. Answer books must be handed to the supervisor-in-charge promptly when the signal is given. Failure to comply with this regulation will be cause for rejection of an answer paper.
11. If a student becomes ill or receives word of domestic affliction during the course of an examination, he/she should report at once to the Supervisor, hand in the unfinished paper and request that it be cancelled. Thereafter, if illness is the cause, the student must go directly to University Health Services so that any subsequent application for a deferred examination may be supported by a medical certificate. An application for Deferred Final Examinations must be submitted to the Registrar by the date specified in the University Calendar. Should a student write an examination, hand in the paper for marking, and later report extenuating circumstances to support a request for cancellation of the paper and for another examination, such request will be denied.
12. SMOKING DURING EXAMINATIONS IS STRICTLY PROHIBITED.
13. NO CALCULATORS OR OTHER AIDS.

Question	Total Marks	Actual Marks
1	10	
2	6	
3	7	
4	7	
5	7	
6	7	
7	7	
8	7	
9	7	
10	7	
11	7	
12	7	
13	7	
14	7	
Total	100	

1. Answer T or F only.

- (a) The Fields Medal, the most famous award in mathematics, was introduced by the Canadian mathematician J.C.Fields. T
- (b) As of 2002, the Fields Institute in Toronto is the only mathematical research institute in Canada. F
- (c) The British born mathematician Andrew Wiles astounded the world with his solution, in 1994, of the Riemann Hypothesis. F
- (d) After the extensive development of Euclidean geometry by the Greeks, the Romans followed by laying the foundations of non-Euclidean geometry. F
- (e) Category theory is a development of mathematics in the 20th century. T
- (f) Complex analysis received its initial development in the 19th century. T
- (g) Probability theory received its initial development in the 19th century. F
- (h) Statistics received its initial development in the early part of the 19th century. T
- (i) $\pi = 22/7$ F
- (j) Calculating machines of various types (electrical or mechanical) go back at least 2,000 years. T

2. The command `>convert(58300294,base,5);` is given to MAPLE and the output is `[4,3,2,0,1,1,1,4,4,0]`. Explain what all this means.

It means 58300294 (base 10) = 104411102134 (base 5)

② If we use 43120114401

3. Name three famous mathematicians of the 20th century, and their nationalities.

4. Find $\gcd(195, 364)$.

$$\begin{array}{r} 1 \\ 195 \overline{) 364} \\ \underline{195} \\ 169 \overline{) 195} \\ \underline{169} \\ 26 \overline{) 169} \\ \underline{156} \\ \hline 13 \end{array}$$

-4 if with error

5. Solve the equation $x^2 + 6x + 12 = 0$, giving the solutions for x in simplest possible form.

$$\begin{aligned}x &= \frac{-6 \pm \sqrt{36-48}}{2} = \boxed{-3 \pm i\sqrt{3}} \quad (7) \\ &= \frac{-6 \pm \sqrt{-12}}{2} \quad (4)\end{aligned}$$

6. Lucky Lennie solves the inequality

$$5 < \frac{x+1}{x+3}$$

by the following steps:

$$5(x+3) < x+1 \Rightarrow 5x+15 < x+1 \Rightarrow 4x < -14,$$

giving $x < -7/2$ as his final solution. Unfortunately his luck has run out and this solution is incorrect.

- (3) (a) Show, by an explicit example, that the solution is incorrect.

- (4) (b) Where was the error, and explain what the error was.

Error in first step since $x+3$
not nec. positive

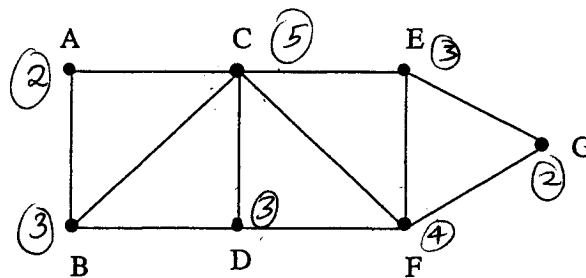
not complete argument byt choose 1st step -1

7. Choose either topic (a) or topic (b) (not both) and write 1-2 paragraphs giving your opinion. You may choose to support or to oppose the statement.

(a) As society and technology progress into the 21st century, it will be more and more important for people to have an increasing understanding of mathematics and statistics.

(b) Thanks to modern calculators, students in elementary school don't have to be taught basic arithmetic (i.e. how to do basic operations, addition and multiplication tables, etc.) any longer.

8. Either find an Euler tour in the following graph, or show that such an Euler tour is impossible.



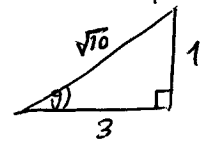
Impossible, 4 odd vertices
 (4) (3)
 4

-1 if say B, D, C, E have an odd # of vertices

9. (a) Given that $\sin \theta = 1/\sqrt{10}$, determine both $\cos \theta$, $\tan \theta$.

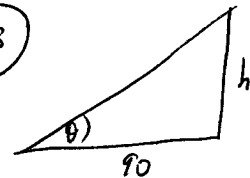
$$\cos \theta = \frac{3}{\sqrt{10}} \quad (2)$$

$$\tan \theta = \frac{1}{3} \quad (2)$$



(b) An observer standing 90 m from the base of a tree notices that the angle subtended from the top of the tree to the horizontal is θ , where θ is the same angle as in (a). Determine the height of the tree.

$$\frac{h}{90} = \frac{1}{3} \Rightarrow \boxed{h = 30 \text{ m}} \quad (3)$$



10. In the following table a, b, c represent the lengths of the sides of a triangle. Match each possibility with one of the following : equilateral triangle, right triangle, isosceles triangle, scalene acute triangle, scalene obtuse triangle, can't exist. The matching should be 1-1.

a b c

5 12 13

10 12 13

5 12 18

12 12 18

5 12 14

12 12 12

right
scalene acute
can't exist
isosceles
scalene obtuse
equilateral

11. Evaluate

$$\lim_{x \rightarrow 3} \frac{x^2 - 8x + 15}{x^2 - 9}$$
$$= \lim_{x \rightarrow 3} \frac{(x-3)(x-5)}{(x-3)(x+3)} = \frac{-2}{6} = \boxed{-\frac{1}{3}}$$

-1 for small errors like missing = or missing lim

12. Mom sends Mary to the store to buy apples, which are 17¢ each, and oranges, which are 22¢ each. If Mary spends exactly \$5.00, how many apples and oranges did she purchase?

$$17x + 22y = 500$$

$$\text{In } \mathbb{Z}_{17}, \quad 5y = 7 \quad (3)$$

$$y = 7 \times 5^{-1} = 7 \times 7 = 15$$

$$x = 10$$

$$\begin{array}{r} 29 \\ 17 \overline{) 500} \\ \underline{34} \\ 160 \\ \underline{153} \\ 7 \end{array}$$

13. (a) Differentiate $f(x) = 5x^4 - x^3 + 6x + 7$.

$$f'(x) = 20x^3 - 3x^2 + 6$$

(-2) if say $f' = f$
or $f = f'$

(-1) if +7 in f'

(b) Differentiate $g(x) = 3e^x + x^{\frac{5}{2}}$.

$$g'(x) = 3e^x + \frac{5}{2}x^{\frac{3}{2}}$$

14. Prove that $\sqrt{5}$ is irrational.

Suppose $\sqrt{5} = \frac{m}{n}$, $m \perp n$ wlog

$$5 = \frac{m^2}{n^2}$$

$$5n^2 = m^2$$

$$\Rightarrow 5|m^2 \Rightarrow m = 5m_1$$

$$\Rightarrow 5n^2 = 25m_1^2$$

$$n^2 = 5m_1^2$$

$$\Rightarrow 5|n^2 \Rightarrow 5|n$$

$$\Rightarrow m, n \text{ not rel prime} \Rightarrow \Leftarrow \square$$

max (2) if incorrect proof