MATH 205 L01 Winter 2002

Midterm 50 Minutes

NAME: KEY ID: _____

No Calculators

1. For each question answer T or F only. [20]

(a) The work of René Descartes had great influence towards unifying the mathematics of his time. T

(b) Grecian mathematics was far more on the theoretical side than the applied side. T

(c) Calculus was one of the great discoveries of Grecian mathematics. F

(d) Probability theory originated with the work of Hindu and Arabic mathematicians in the period 400-900 AD. F

(e) Algebra originated with the work of Hindu and Arabic mathematicians in the period 400-900 AD. T

(f) Chinese mathematicians around 400 AD already knew the value of π to about 6 digits. T

(g) Greek mathematicians not only knew the Earth was round but also calculated its size. T

(h) Leonhard Euler was the most famous Italian mathematician during the 1700's. F

(i) Sophie Germain, in the period 1810-1830, was the first notable female mathematician. F

(j) Prime numbers of the form $2^n - 1$ (where n is itself a prime), are named after the Frenchman Pierre de Fermat. F

2. To 3 decimal place accuracy it is true that $\sqrt{2} = 1.414$. Explain, in one to two sentences, whether the statment $\sqrt{2} = 1.41414141...$ is true or false. [20]

False

because $\sqrt{2} \notin \mathbb{Q}$, whereas $1.\overline{41} \in \mathbb{Q}$

3. (a) In base 3, find $1202 + 2211$. [20]

$$\begin{array}{r} 1202 \\ 2211 \\ \hline 11120 \end{array}$$

$= \boxed{11120}$

(6)

(b) In base 2, find 1011×1101.

$$\begin{array}{r} 1011 \\ 1101 \\ \hline 1011 \\ 101100 \\ 1011000 \\ \hline 10001111 \end{array}$$

$\boxed{10001111}$

(6)

(c) Write the base 10 number 5,376 as a Mayan number.

$20\,)\,5376$

$20\,)\,268$ 16

13 8

(8)

4. (a) In $\mathbb{Z}_{23}$, find $16(3^2+4^2)$. [20]

$= 16 \times 2 = \boxed{9}$

(6)

(b) In $\mathbb{Z}_{23}$, find 7^{-1}.

1 24 47 70 . . .

$70 = 7 \times 10$

$7^{-1} = \boxed{10}$

(7)

(c) If 28^{2244} is divided by 23, what is the remainder?

$28^{2244} = 5^{2244}$ in $\mathbb{Z}_{23}$

$= 5^0$ by Fermat's Little Theorem, since $5^{22} = 1$ and $22 | 2244$

$= \boxed{1}$

(7)

5. (a) Find the greatest common divisor gcd(182, 403). [20]

$403 = 2 \cdot 182 + 39$, $182 = 4 \cdot 39 + 26$, $39 = 1 \cdot 26 + 13$, $26 = 2 \cdot 13 + 0$

$= \boxed{13}$

(15)

(b) Using (a) explain why the Diophantine equation

$$182x + 403y = 26,005$$

can have no solution (in integers).

Reason $13 \nmid 26,005$

(clearly $13 \mid 26,000$, hence $26005 \equiv 5 \pmod{13}$)

(5)