MATH 205   L01   W 2004

FINAL EXAMINATION
3 Hours

NAME: _____ **ID:** _____

A standard size formula sheet is allowed, and no other aids.

1. (a) MAPLE is given the command >ifactor(290). The output will be:

(b) One wishes to use MAPLE to determine the period of 1/157 as a repeating decimal. Which command(s) would you use, and explain how you would then solve the question.

2. Give the names of three famous mathematicians of ancient Greece, and mention one of their achievements (for any one of the three).

3. For each of the following answer True or False.

   (a) The graph of any linear equation will intersect the x-axis in exactly one point. _____

   (b) The graph of any linear equation will intersect the y-axis in exactly one point . _____

   (c) The graph of any quadratic equation will intersect the x-axis in exactly two points. _____

   (d) The graph of any quadratic equation will intersect the y-axis in exactly two points . _____

   (e) Given the three points $P = (1, 4)$, $Q = (3, 2)$, $R = (4, 6)$, there will be a unique quadratic function $y = ax^2 + bx + c$ passing through P, Q, R. _____

   (f) A graph in which the degree of every vertex is even will always have a Hamiltonian circuit. _____

   (g) A graph in which the degree of every vertex is even will always have an Euler circuit. _____

   (h) Mathematicians and computing scientists have calculated π to some 50,000,000,000 decimal places in order to see whether or not it is irrational. _____

   (i) The two most famous French mathematicians in the 20th century were Lagrange and Laplace. _____

   (j) The axiomatic approach to mathematics, first developed by Euclid, regained its importance towards the end of the 19th century. _____

4. A 5-regular graph has 30 vertices.

   (a) Determine how many edges it will have.

   (b) Show whether or not this graph admits an Euler path.

5. Write one to two paragraphs supporting either statement (a) or statement (b).

   (a) Computers will have little influence in mathematics besides speeding up long computations.

   (b) Computers have already had an important influence in mathematics, far beyond just extra computational power, and this influence will continue to grow in the future.

6. It is known that, in a certain brewing process, adding 6 grams of yeast will lead to an output of 9 barrels of beer, while adding 8 grams of yeast gives output 10 barrels.

   (a) Determine a linear relation (in slope-intercept form) that fits the given data.

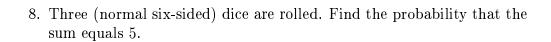   (b) What would the output be if 9 grams of yeast are added.

7. (a) Convert 799_{10} to base 5.

(b) Convert 1011011010011_2 to base 8.

(c) Convert the Mayan number below to base 10.

8. Three (normal six-sided) dice are rolled. Find the probability that the sum equals 5.

9. For the graph below, find an Euler circuit (if possible) and a Hamilton circuit (if possible).

10. In the field $\mathbb{Z}/53$, determine 41^{-1}.

11. In the field $\mathbb{Z}/7$, the perfect squares are $0, 1, 2, 4$. Find the set of perfect squares in each of the fields $\mathbb{Z}/3$, $\mathbb{Z}/5$, $\mathbb{Z}/11$, and use inductive reasoning to formulate a conjecture as to how many perfect squares there are in the field $\mathbb{Z}/p$, for any odd prime p.

12. Prove that $\sqrt{2} + \sqrt{5}$ is irrational. You may assume (as proved in class) that $\sqrt{n}$ is irrational for any natural number n that is not a perfect square.