MATH 205 L01 W 2004
MIDTERM 50 Minutes

NAME: ______________________ **ID**: ________

1. Each of the following numbers is composite. For each one, find a factor (you do not have to factor the number completely). [10]

   (a) 6,111,003

   (b) 6,111,005

   (c) 7,121,829

   (d) $2^{35} - 1$

   (e) $2^{2^5} + 1$ [Hint : remember Euler]

2. For each of the following answer True or False. [20]

   (a) The difference of two natural numbers is always an integer. ________

   (b) The difference of two integers is always a natural number. ________

   (c) In the modular system $\mathbb{Z}/n$, subtraction is always possible. ________

   (d) In the modular system $\mathbb{Z}/n$, division is always possible. ________

   (e) The Fundamental Theorem of Arithmetic states that for any two natural numbers m, n, there exist integers $q \geq 0$, $0 \leq r < m$ such that $n = qm + r$. ________

   (f) All numbers $2^p - 1$, where p is prime, are themselves prime. ________

   (g) All numbers $2^{2^n} + 1$, $n \geq 0$, are prime. ________

   (h) The first known proof that there are infintely many prime numbers is due to Euclid. ________

   (i) The Mayan number system uses only three symbols. ________

   (j) Canada has three mathematics institutes. ________

3. Consider the sequence $F_0 = 0, F_1 = 1, F_2 = 1, F_3 = 2, F_4 = 3,$ $F_5 = 5, \ldots.$. [20]

(a) Write out the next 7 terms of this sequence.

$$F_6 = \underline{\qquad\quad}, F_7 = \underline{\qquad\quad}, F_8 = \underline{\qquad\quad}, F_9 = \underline{\qquad\quad},$$

$$F_{10} = \underline{\qquad\quad}, F_{11} = \underline{\qquad\quad}, F_{12} = \underline{\qquad\quad}.$$

(b) This famous sequence is named the ________________ sequence.

(c) Using inductive reasoning and the values $F_0, \ldots, F_{12}$ as above, make a plausible statment about when F_n is even, i.e. for which values of n will F_n be even.

(d) Prove your statement in (c), using deductive reasoning.

4. (a) Find $\gcd(42, 303)$ by factoring the two numbers. [20]

(b) Find gcd(4403,2686) by any method you wish.

5. In $\mathbb{Z}/31$, (a) Find $7^2 + 8^2$. [20]

(b) Find 4^{-1}.

(c) Solve the equation $4x + 7 = 2$.

6. Carry out the Mayan addition: [10]