MATH 205   L01   W 2005

# FINAL EXAMINATION SOLUTIONS

1. This is a matching question. Match each property (i)-(vi) with one of the graphs (a)-(f), and give answers in the box below.

   (i) This graph is C_6.
   (ii) This graph is a tree.
   (iii) This graph has an Euler circuit but no Hamilton circuit.
   (iv) This graph has a Hamilton circuit but no Euler circuit.
   (v) This graph is not connected.
   (vi) This graph is non-planar.

   Remark: The graph (e) should have an extra edge drawn in, from the extreme left hand vertex to the extreme right hand vertex, and graph (c) should have an extra edge drawn in from the lower middle vertex to the upper right vertex.

(i)	(ii)	(iii)	(iv)	(v)	(vi)
f	d	e	b	a	c

(a)

(b)

(c)

(d)

(e)

(f)
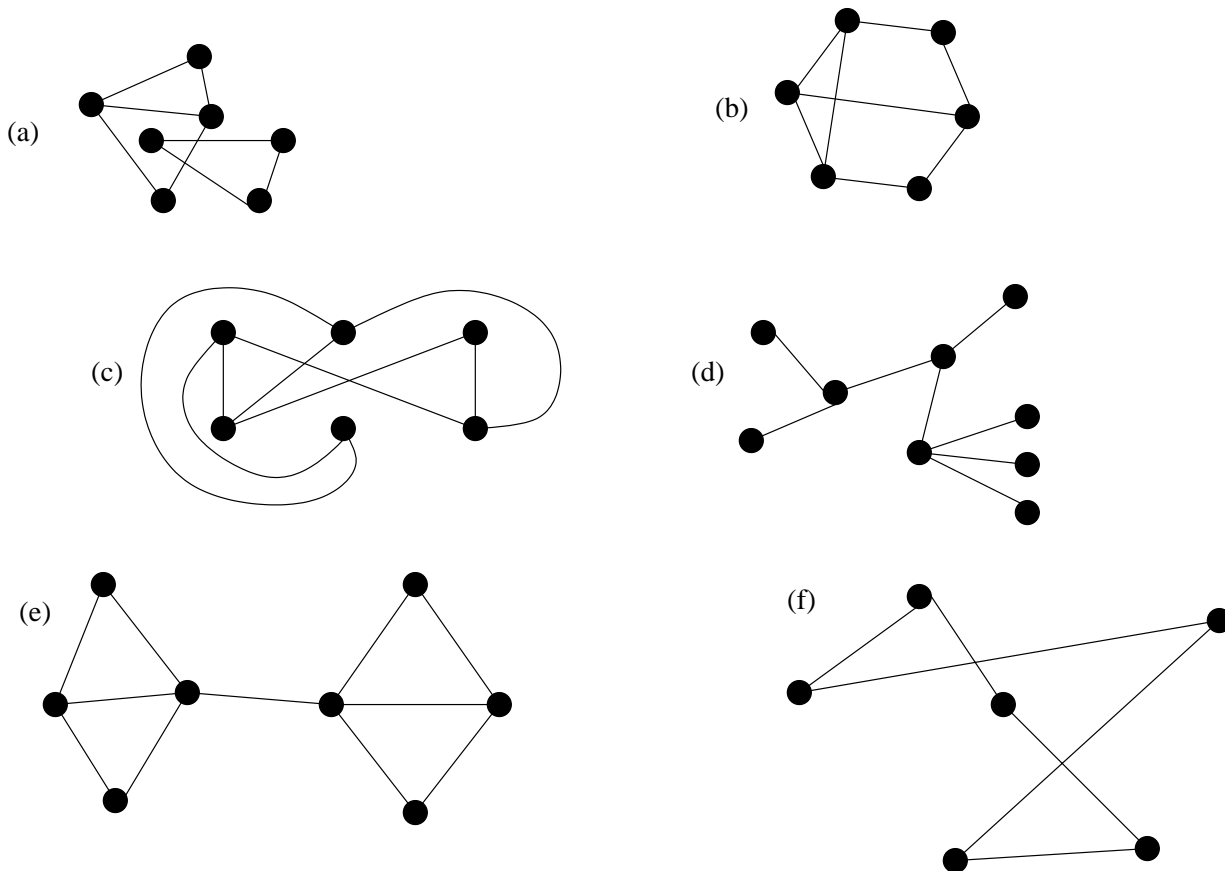
2. Choose a nation and then name three famous mathematicians associated with that nation, by birth or by residence. Also state an important achievement for any one of the three mathematicians you have chosen, as well as the century he or she lived in.

Many answers possible. Sample answer is France: Fermat (17th), Germain (19th), Galois (19th). Germain is famous for Sophie Germain primes and her work on Fermat's Last Theorem.

3. (a) Which command or commands would you use in MAPLE to find $20^{1/3}$, the cube root of 20, to 50 digits?

evalf(20∧(1/3),50)

(b) Show the output MAPLE would give for the command

$> f := 2 * x \wedge 3 - 3 * x \wedge 2 + 15 * x + 1 \, ;$

$f := 2x^3 - 3x^2 + 15x + 1$

(c) Describe the output MAPLE would give for command (b) followed by

$> \mathrm{plot}(f, x = 0 \, .. \, 4);$

It would plot the function f for $0 \le x \le 4$.

4. For each of the following answer True or False. Do not use T or F.

Remark: Questions (c), (d), (e) irrelevant for 2006.

(a) $\pi = 22/7$.   False

(b) 4.99999.... = 5.   True

(c) The graph of any cubic polynomial will intersect the x-axis in one, two, or three points.   True

(d) The graph of any quadratic polynomial will intersect the y-axis in exactly one point.   True

(e) Given the two points $P = (1,5)$, $Q = (4,-1)$, there will be a unique linear function passing through P and Q.   True

(f) The radius of the Earth was first determined by Sir Isaac Newton.   False

(g) The complete graph K_n always has an Euler circuit.   False

(h)   John Nash was one of the first American mathematicians to win the Fields Medal.   False

(i)   In 1900 Hilbert presented a list of 23 of the most important questions in mathematics, and to date none of these has been solved.   False

(j)   The conflict between doing mathematics for its own sake, and doing it for its practical applications, is a relatively recent development in the 20th century.   False

5. (a) Write $2431_{(5)}$ as a base 10 number.

366

(b)   Write the Roman numeral MMMDCCXLIV   as a base 10 number (remember I=1, V=5, X=10, L=50, C=100, D=500, M=1000).

3,744

(c)   Write the base 10 number 2005 as a number in base 3.

2202021

6. About eight years ago a President of the University of Calgary, who has a Ph.D. in Sociology, said in his inaugural address that "personally, in all the academic work I have done, I never had any need for mathematics beyond Grade 10." Write one to two paragraphs supporting either statement (a) or statement (b). Try to back up your arguments with as many facts as possible.

(a)   This is a reasonable statement for an important speech by a university president.

(b)   This is an extremely bad statement for such an important speech.

Personally I would choose statement (b), however either statement could be chosen as long as the argument is clear and supported by a few facts.

7. (a) Complete the given Mayan addition.

(Mayan numeral) + (Mayan numeral) = .

The answer is

(Mayan numeral)

(b) In the base 3, find 2021×121.

1100011

8. Apples cost 11¢ each and oranges cost 31¢ each. If Billy goes to the store and spends a total of \$7.10 buying apples and oranges, how many did he buy of each, given also that he bought more oranges than apples?

Answer: 11 apples, 19 oranges

9. (a) In the field $\mathbb{Z}/71$, determine 42^{-1}.

Answer : $42^{-1} = 22$

(b) Write the repeating decimal $.\overline{135}$ as a rational number in the field $\mathbb{Q}$.

$$\frac{5}{37}$$

5

10. There is a field with 4 elements, which are written 0, 1, x, y. Complete the addition and multiplication tables for this field.

+	0	1	x	y
0	0	1	x	y
1	1	0	y	
x	x	y	0	
y	y			0

×	0	1	x	y
0	0	0	0	0
1	0	1	x	y
x	0	x	y	
y	0	y		

Answer: The missing elements, inserting from left to right and top to bottom, are $x, 1, x, 1$ in the first table, and $1, 1, x$ in the second table.