MATH 205 L01 W 2004 MIDTERM AND SOLUTIONS 50 Minutes

NAME: ____________________ **ID**: ________

Remark for 2006 : Questions with a * are not applicable for this year's review.

1. Each of the following numbers is composite. For each one, find a factor (you do not have to factor the number completely). [10]

   (a) 6,111,003 3 is a factor

   (b) 6,111,005 5 is a factor

   (c) 7,121,829 3 and 11 are factors

   (d) $2^{35} - 1 := (2^5)^7 - 1 = 32^7 - 1^7 = (32-1)(32^6 + 32^5 + 32^4 + 32^3 + 32^2 + 32 + 1)$, thus $32 - 1 = 31$ is a factor

   (e) $2^{2^5} + 1$ [Hint : remember Euler] Euler discovered that 641 is a factor

2. For each of the following answer True or False. [20]

   (a) The difference of two natural numbers is always an integer. True

   (b) The difference of two integers is always a natural number. False

   (c*) In the modular system $\mathbb{Z}/n$, subtraction is always possible. True

   (d*) In the modular system $\mathbb{Z}/n$, division is always possible. False

   (e) The Fundamental Theorem of Arithmetic states that for any two natural numbers m, n, there exist integers $q \geq 0$, $0 \leq r < m$ such that $n = qm + r$. False

   (f) All numbers $2^p - 1$, where p is prime, are themselves prime. False

(g) All numbers $2^{2^n}+1$, $n \geq 0$, are prime. False

(h) The first known proof that there are infinitely many prime numbers is due to Euclid. True

(i*) The Mayan number system uses only three symbols. True

(j) Canada has three mathematics institutes. True

3. Consider the sequence $F_0 = 0, F_1 = 1, F_2 = 1, F_3 = 2, F_4 = 3, F_5 = 5, ...$. [20]

(a) Write out the next 7 terms of this sequence.

$$F_6 = 8, F_7 = 13, F_8 = 21, F_9 = 34, F_{10} = 55, F_{11} = 89, F_{12} = 144.$$

(b) This famous sequence is named the Fibonacci sequence.

(c) Using inductive reasoning and the values $F_0, ..., F_{12}$ as above, make a plausible statment about when F_n is even, i.e. for which values of n will F_n be even. Answer : F_n is even iff n is divisible by 3.

(d) Prove your statement in (c), using deductive reasoning. Look at the sequence modulo 2 (i.e. even or odd). The pattern is even,odd,odd,even,odd,odd, etc. This pattern must continue since even+odd = odd, odd+odd=even, odd+even=odd. Thus every third Fibonacci number will be even, starting with $F_0 = 0$, proving that F_n is even iff $n = 0, 3, 6, 9, 12, ..$, i.e. iff n is divisible by 3.

4. (a) Find $\gcd(42, 303)$ by factoring the two numbers. [20]
$42 = 2 \times 3 \times 7$, $303 = 3 \times 101$. So the gcd = 3.

(b) Find gcd(4403,2686) by any method you wish.
Euclidean algorithm best method, answer is gcd = 17