# MATH 205 L01 W 2006
# MIDTERM REVIEW + ANSWERS

1. Give the next number in each sequence.

   (a) 1 2 3 4 5 10 11 12 13 14 15 20 21 (counting in base 6)

   (b) 1 2 5 10 17 26 37 $(n^2 + 1)$

   (c) 2 4 6 11 13 15 20 22 (even numbers in base 7)

   (d) 1 1 2 3 5 8 13 21 34 (Fibonacci sequence)

   (e) 1 1 3 7 17 41 99 (rule is $x_{n+2} = 2x_{n+1} + x_n$)

   (f) 1 2 9 28 65 126 217 (take the successive differences, and then the successive differences of these)

2. Inductive or deductive reasoning:

   (a) The past 6 years your income tax has averaged 31%. You conclude that this year it should be nearly 31%. Inductive

   (b) Five suspects for a certain crime are eliminated and altogether there were 6. You conclude that the criminal must be the remaining suspect. Deductive

3. Using mathematical induction, prove $\sum_{j=1}^{n} j = n(n+1)/2$.

   (done in lecture)

4. Give the names of three famous twentieth century mathematicians. Do the same for the nineteeth century. In each case also give the nationality.

   (many have been given in the lectures)

5. Consider the sequence $F_0 = 0, F_1 = 1, F_2 = 1, F_3 = 2, F_4 = 3, F_5 = 5, \ldots$ .

   (a) Write out the next 7 terms of this sequence.

   8,13,21,34,55,89,144

   (b) This famous sequence is named the _____sequence.

   Fibonacci

   (c) Using inductive reasoning and the values $F_0, ..., F_{12}$ as above, make a plausible statement about when $F_n$ is divisible by 4, i.e. for which values of $n$ will $F_n$ be divisible by 4.

Solution: Note that $F_0, F_6, F_{12}$ are the only ones divisible by 4 on the list. A reasonable conclusion is that $F_n$ is divisible by 4 iff $n$ is divisible by 6.

(d) Show that $F_n + F_{n+3} = 2F_{n+2}$, for example (for $n = 2$) $1 + 5 = 2 \times 3$.

[Hint : You may assume the basic defining relation of this sequence $F_{n+1} = F_n + F_{n-1}$. Show then that $F_{n+2} = 2F_n + F_{n-1}$, derive a similar formula for $F_{n+3}$, and use these to prove the theorem.]

We just show most of the proof. $F_{n+2} = F_{n+1}+F_n = (F_n+F_{n-1})+F_n = 2F_n+F_{n-1}$. Similarly you should derive $F_{n+3} = 3F_n+2F_{n-1}$, using the previous formula for $F_{n+2}$. Then $F_{n+3} + F_n = 4F_n + 2F_{n-1} = 2(2F_n + F_{n-1}) = 2F_{n+2}$.

6. Use deductive reasoning to show that the answer to the "trick" given on p.76-45 in the text, is always 3.

   Solution : Start with $n$, then the successive steps give $3n$, $3n+9$, $n+3$, $n+3-n=3$.

7. Solve the equation $34x+10y = 1000$, where $x, y$ are positive integers.

   $x = 25, y = 15$

8. Complete the addition and multiplication tables below for base 5 arithmetic. Then carry out the operations, all in base 5, of
   $31241 - 13042$, $234 \times 32$.

| + | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| 1 | 2 | 3 | 4 | 10 |
| 2 | 3 | 4 | 10 | |
| 3 | 4 | 10 | | |
| 4 | 10 | 11 | | 13 |

| × | 2 | 3 | 4 |
|---|---|---|---|
| 2 | 4 | 11 | 13 |
| 3 | 11 | | |
| 4 | 13 | | 31 |

   $31241 - 13042 = 13144$, $234 \times 32 = 14143$

9. A graph has 7 vertices with degrees 3,4,4,5,6,6,8.

   (a) How many edges does it have?

   18 edges

   (b) Does it admit an Euler path or Euler circuit?

   It admits an Euler path, not an Euler circuit

10. Draw the planar graph with vertices $\{a, b, c, d, e, f\}$ and edges $\{ab, ad, ae, bd, bc, bf, ce, cf, de, df, ef\}$. Find an Euler path, Hamilton cycle, and also colour with 4 colours.

    (not done here)

11. Show the diophantine equation $585x + 741y = 92$ has no solution.

    Using the Euclidean algorithm, we find gcd(585,741) = 39. Since 39 is not a divisor of $92 = 2^2 \times 23$, there can be no solution.

12. Write 1043 in base 6.

    $1043_{10} = 4455_6$

13. Find all primes between 320 and 340.

    Use the sieve, checking 2,3,5,7,11,13,17. The remaining primes are 331 and 337

14. Factor 54417.

    $54417 = 3 \times 11 \times 17 \times 97$

15. Use inductive reasoning to find the remainder if $5^{1000}$ is divided by 7.
    remainder = 2