MATH 205 L01 W 2006

FINAL EXAMINATION
3 Hours

NAME: ______________________________ **ID**: ________

A standard size formula sheet is allowed, and no other aids.

1. For each of the following questions circle T (true) or F (false) in the table below.

   (a) A connected graph in which each vertex has degree 6 admits an Euler circuit.

   (b) $\pi = \dfrac{22}{7}$.

   (c) $4.9999... < 5$

   (d) The sum of two rational numbers is always rational.

   (e) The sum of two irrational numbers is always irrational.

   (f) The mathematics of Ancient Greece concentrated completely on Geometry.

   (g) Mathematicians and computer scientists have calculated π to about 50,000,000,000 decimal places in order to see whether or not it is irrational.

   (h) The work of certain mathematicians such as Bolyai, Lobachevsky, and Gödel has had major impact on philosophy.

   (i) The algebraic system $\mathbb{Z}_{211}$ is a field.

   (j) The Fields Medals are named after a Canadian mathematician, John Charles Fields.

Question	*(a)*	*(b)*	*(c)*	*(d)*	*(e)*	*(f)*	*(g)*	*(h)*	*(i)*	*(j)*
	T	*T*	*T*	*T*	*T*	*T*	*T*	*T*	*T*	*T*
	F	*F*	*F*	*F*	*F*	*F*	*F*	*F*	*F*	*F*

2. (a) Which command or commands would you use in MAPLE to find π^5, to 50 digits?

   (b) Show the output MAPLE would give for the command

   $f := 2 * x \wedge 3 - 3 * x \wedge 2 + 15 * x + 1\,;$

   (c) What output would MAPLE give for the command
   ifactor(408); ?

3. Using base 6 arithmetic, multiply the numbers 352×413 (all numbers in base 6).

4. Write a paragraph supporting either opinion (a) or opinion (b) about the influence of computers in mathematics.

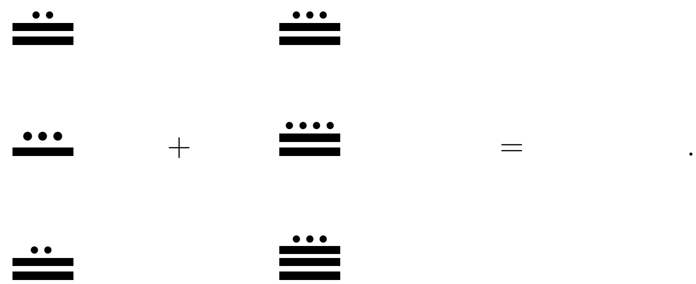
   (a) Computers will have little influence in mathematics besides speeding up long computations.

   (b) Computers have already had far reaching influence in mathematics, far beyond just extra computational power, and this influence will continue to grow in the future.

5. Let $\mathcal{G}$ be a graph that is a tree. Use mathematical induction to prove that the number of vertices minus the number of edges equals 1, i.e. $V_{\mathcal{G}} - E_{\mathcal{G}} = 1$.

6. Solve the equation $\quad 20x = 3 \quad$ in $\mathbb{Z}_{73}$.

7. (a) Complete the given Mayan addition.



(b) Convert the base 8 (octal) number 6705314 to a base 2 (binary) number.

8. (a) Simplify the rational number $(\frac{2}{5} - \frac{8}{3}) \div (\frac{1}{3} - \frac{3}{5})$.

(b) Write the repeating decimal $0.\underline{621}$ as a rational number, simplifying to lowest terms.

9. (a) Consider primes p of the form $p = 3m + 1$, such as $p = 7, 13, 19, 31, 37, 43, \ldots$. We would like to know many non-zero numbers in $\mathbb{Z}_p$ are perfect cubes? For example, in $\mathbb{Z}_{19}$, the non-zero perfect cubes are $1, 7, 8, 11, 12, 18$ (e.g. $1^3 = 1, 2^3 = 8, 3^3 = 8, 4^3 = 7$, etc.). Make the same calculation for a couple of other primes of this form, for example at least for $p = 7, 13$, and then use inductive reasoning to make a conjecture about how many perfect cubes there will be in general.

   (b) Is the conjecture you made in (a) true only for primes of the form $p = 3m + 1$, or is it true for any prime $p > 3$? Answer yes or no, and explain.

10. Some parts of mathematics have had the effect of unifying much work that came before it. Name two mathematicians, whose work unified and/or synthesized much previous work, and also specify what the contribution of each mathematician was.