MATH 205 NOTES ON Z-PROCESS

Our goal in this example is to solve the Diophantine equation $89x + 109y = 2000$ with positive integers.

1. Euclidean Algorithm

```
      1
 89)109
     89   4
    20)89
        80  2
        9)20
           18  4
           2 )9
               8
               1
```

2. Now comes the Z-process. Notice that the first row is formed from the quotients in the Euclidean algorithm process, from last (bottom) to first (top). The second row starts with 0, 1, then successively multiply and add, e.g. $9 = 2 \times 4 + 1$, $40 = 4 \times 9 + 4$, etc.

		4	2	4	1
0	1	4	9	40	49
$-$	$+$	$-$	$+$	$-$	$+$

3. We now know, from (2.), that $49 \times 89 + (-40) \times 109 = 1$. Multiplying this by 2000 gives

$$98000 \times 89 + (-80000) \times 109 = 2000,$$

$$(98000 - 109s) \times 89 + (-80000 + 89s) \times 109 = 2000 \ .$$

By long division we find $98000 = 109 \times 899 + 9$. This means that we should take $s = 899$. Substituting this value for s gives the solution $89 \times 9 + 109 \times 11 = 2000$, which is the desired solution. We remark that in this particular example there was a unique solution in positive integers, but this need not always be the case - there may be no solution or more than one (in positive integers) in general.