# Practice Problems S3

1. Determine whether the following matrices are elementary matrices or not; write down the inverses of the elementary matrices (explain your answer):

   (a) $\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$, (b) $\begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix}$, (c) $\begin{bmatrix} 1 & 2 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$, (d) $\begin{bmatrix} 1 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$,

   (e) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix}$, (f) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 2 \\ 0 & 0 & -1 \end{bmatrix}$.

2. Find an invertible matrix $U$ such that the product $R = UA$ is the reduced row-echelon form of $A$ if

$$A = \begin{bmatrix} 1 & -1 & 3 & 5 \\ 3 & -2 & 1 & -2 \\ -1 & 1 & 1 & 3 \end{bmatrix}.$$

3. Express the following matrix as a product of elementary matrices:

$$A = \begin{bmatrix} 5 & 3 \\ 2 & 1 \end{bmatrix}.$$

4. Find the matrix of the reflection in the line $y = -x$.

5. Find a rotation or a reflection that is equal to

   (a) reflection in the $y$-axis followed by rotation through $\pi/2$;

   (b) rotation through $\pi/2$ followed by reflection in the line $y = x$.

6. Given $T([1 \ -2]^T) = [3 \quad 4]^T$ and $T([-2 \quad 5]^T) = [-1 \quad 4]^T$, find $T([-4 \quad 3]^T)$ if $T$ is a linear transformation.

7. Consider a Markov chain that starts in state 1 with transition matrix $P = \begin{bmatrix} \frac{1}{3} & \frac{2}{3} \\ \frac{2}{3} & \frac{1}{3} \end{bmatrix}$.

   (a) Explain why the chain is regular.

   (b) Find the probability that the chain is in state 1 after 2 transitions.

   (c) Find the steady-state vector for the chain.

**Recommended Problems:**

Pages 68 - 69: 1; 2a, b; 3a; 5a, b; 6 a,b; 7; 8b, c;
Pages 80-81: 1. b, c; 2. a; 3, 4, 5, 9, 10, 12; Pages 101-102: 1, 2, a, c;
Page 101-102: 1. a, b, f, g, h, k, l, m, n, o, p; 5. a, b; 6, 7, 8, 9, 11, 13, 14, 15;