Practice Problems S4

1. By inspection, find the determinants of the following matrices:

(a) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$; (b) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & -3 & 0 \\ 0 & 0 & 1 \end{bmatrix}$; (c) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 3 & 0 & 1 \end{bmatrix}$;

(d) $\begin{bmatrix} 1 & -2 & 3 \\ 2 & 1 & 4 \\ 2 & -4 & 6 \end{bmatrix}$; (e) $\begin{bmatrix} 1 & 0 & 4 & 9 \\ -8 & -7 & 12 & 2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 5 & 3 \end{bmatrix}$.

2. Compute the determinants of the following matrices

(a) $A = \begin{bmatrix} -2 & 1 & 3 \\ 1 & -7 & 4 \\ -2 & 1 & 3 \end{bmatrix}$; (b) $A = \begin{bmatrix} 3 & 5 & -2 & 6 \\ 1 & 2 & -1 & 1 \\ 2 & 4 & 1 & 5 \\ 3 & 7 & 5 & 3 \end{bmatrix}$.

3. Find the inverse of $A = \begin{bmatrix} -1 & 0 & 1 \\ 1 & 1 & 0 \\ -1 & -1 & 1 \end{bmatrix}$ using the adjoint formula.

4. Given $A = \begin{bmatrix} 3 & -1 & 2 \\ 5 & 5 & -2 \\ 1 & 2 & 3 \end{bmatrix}$, find the $(1,3)$-entry of A^{-1}.

5. If A and B are 4×4-matrices with $\det(A) = -2$ and $\det(B) = 2$, find

$$\text{(a)} \quad \det\left(\text{adj}(A)B^T A^4 (B^2)^{-1}\right); \quad \text{(b)} \quad \det\left(A^3 (B^2)^T \left((\text{adj}(A))^{-1}\right)^3 B^{-1}\right).$$

6. Let $A = \begin{bmatrix} a & b & c \\ 1 & -1 & 2 \\ d & e & f \end{bmatrix}$, $B = \begin{bmatrix} a & b & c \\ 3 & -2 & 1 \\ d & e & f \end{bmatrix}$ and $C = \begin{bmatrix} a & b & c \\ 1 & 0 & -3 \\ d & e & f \end{bmatrix}$ be 3×3-matrices. If $\det(A) = 4$ and $\det(B) = 5$, find $\det(C)$.

7. For which values of $c \in \mathbb{R}$ is $A = \begin{bmatrix} 1 & c & 0 \\ 2 & 0 & c \\ c & -1 & 1 \end{bmatrix}$ invertible?

8. Solve the following system by Cramer's rule:
(a) $\begin{cases} x + 2y = 4 \\ 3x + 7y = 13 \end{cases}$; (b) $\begin{cases} 3x - 2y + 4z = -3 \\ 5x + 3y + z = 0 \\ 2x + 6y - 5z = 6 \end{cases}$.

Recommended Problems:

Pages 114-115: 1. a, b, f, g, h, k, l, m, n, o, p; 5. a, b; 6, 7, 8, 9, 11, 13, 14, 15;
Pages 126-127: 1. a, c; 2. a, b, c, d; 3, 4, 6, 8, 9, 10.