Practice Problems S5

1. Let A be an $n \times n$ matrix and $0 \neq k \in \mathbb{R}$. Prove that $\lambda \in \mathbb{R}$ is an eigenvalue of A if and only if $k\lambda$ is an eigenvalue of $k\,A$.

2. By inspection, find the eigenvalues of the following matrices:

   (a) $A = \begin{bmatrix} 3 & 1 & 4 \\ 0 & -2 & 2 \\ 0 & 0 & 5 \end{bmatrix}$; (b) $B = \begin{bmatrix} -2 & 0 & 0 \\ 1 & 2 & 5 \\ 4 & 0 & 4 \end{bmatrix}$.

3. Compute $P^{-1}AP$ and then A^n if $A = \begin{bmatrix} 6 & -5 \\ 2 & -1 \end{bmatrix}$ and $P = \begin{bmatrix} 5 & 1 \\ 2 & 1 \end{bmatrix}$.

4. Diagonalize the following matrices (i.e., find an invertible matrix P such that $P^{-1}AP$ is a diagonal matrix if

   (a) $A = \begin{bmatrix} -4 & 1 & 4 \\ -2 & 1 & 2 \\ -3 & 1 & 3 \end{bmatrix}$; (b) $A = \begin{bmatrix} 3 & 1 & 1 \\ -4 & -2 & -5 \\ 2 & 2 & 5 \end{bmatrix}$.

5. For which values of k does the matrix $A = \begin{bmatrix} 2 & 3 \\ k & 4 \end{bmatrix}$ have an eigenvalue of multiplicity 2.

6. Determine whether the following matrices are diagonalizable or not:

   (a) $A = \begin{bmatrix} 1 & 2 \\ 3 & -4 \end{bmatrix}$; (b) $B = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$; (c) $C = \begin{bmatrix} 1 & 0 & 3 \\ 2 & 1 & 2 \\ 0 & 0 & 2 \end{bmatrix}$.

7. Solve the following linear recurrences:

   (a) $x_{k+2} = 2x_k - x_{k+1}$, where $x_0 = 1$ and $x_1 = 2$;

   (b) $x_{k+3} = -2x_k + x_{k+1} + 2x_{k+2}$, where $x_0 = 1$ and $x_1 = 2 = x_2$.