Practice Problems S6 (Complex Numbers)

1. Write the following complex numbers in the form $a + bi$:

   (a) $\frac{3-i}{2i+5}$, (b) $(2-3i)^3$, (c) $\frac{1-i}{2-3i} - \frac{1+2i}{5+i}$, (d) $e^{5i\pi/3}$.

2. Express the following complex numbers in polar from: (a) $(1-\sqrt{3}i)^5$, (b) $(\sqrt{3}-i)(2-2i)$, (c) $-2e^{\pi i/3}$

3. Prove that $\cos(\theta_1+\theta_2) = \cos(\theta_1)\cos(\theta_2) - \sin(\theta_1)\sin(\theta_2)$ and $\sin(\theta_1 + \theta_2) = \cos(\theta_1)\sin(\theta_2) - \sin(\theta_1)\cos(\theta_2)$.

4. (a) Express the number $z = (1-i)(-1+\sqrt{3}i)$ in polar form and in the form $a + bi$;

   (b) Find $\cos(5\pi/12)$ and $\sin(5\pi/12)$.

5. Solve the following equations:

   (a) $(i+z) - 3i(2-z) = iz + 1$;

   (b) $z(1+i) = \bar{z} - (3+2i)$;

   (c) $3x^2 + 5x + 10 = 0$;

   (d) $z^2 = -15 - 8i$;

   (e) $z^2 - (3-2i)z + (5-i) = 0$.

6. Solve the following system of linear equations:

$$\begin{cases} x + iy - iz & = 3+i \\ -ix + 2y + iz & = 2 \\ (i-1)x - (1+2i)y + 2z & = i-1 \end{cases}.$$

7. Find the inverse of $A = \begin{bmatrix} 1 & 1-i \\ 2+i & 3+i \end{bmatrix}$.

8. Diagonalize the matrix $A = \begin{bmatrix} 1 & i \\ i & 1 \end{bmatrix}$.

9. Find the 8th roots of $z = 128(-1 - \sqrt{3}i)$.

Recommended Problems:

Pages 482 - 483: 1, 2, 3 a, 4 a, b; 5 a, b, c; 6 a, b, d; 10 a, b;11 a, b, c; 18 , 19, 23.