Practice Problems S7 (Vector Geometry)

1. Let $P_1(2, 1, -2)$ and $P_2(1, -2, 0)$ be points in $\mathbb{R}^3$.
   (a) Find the parametric equations of the line through P_1 and P_2;
   (b) Find the coordinates of the point P that is 1/4 of the way from P_1 to P_2.

2. Find the point of intersection P between the lines (if they are concurrent):
$$\begin{cases} x = 3 + t \\ y = 2 + 3t \\ z = -1 - 3t \end{cases} \quad \text{and} \quad \begin{cases} x = 1 - s \\ y = 1 + 2s \\ z = 3 + s \end{cases}.$$

3. Find the equation of the plane passing through the point $P(3, -7, 5)$ and is perpendicular to the line $\begin{cases} x = 2 + 6t \\ y = -5 - 6t \\ z = 3 + 5t \end{cases}$

4. Find the equation of the plane through the points $A(3, -7, 1)$, $B(2, 0, -1)$ and $C(1, 3, 0)$. Check if the point $D(5, 1, 1)$ lies on this plane.

5. Determine whether the plane $2x-3y+z = 1$ contains the line $\begin{cases} x = 3 + 2t \\ y = 2 \\ z = 1 - 4t \end{cases}$.

6. Find the line of intersection of the planes $(\pi_1) \equiv 3x + 5y + 4z = 5$ and $(\pi_2) \equiv x + 2y + 3z = 2$.

7. Find the shortest distance from the point $P(1, 1, 1)$ to the line $\begin{cases} x = 3 + t \\ y = 9 \\ z = 10 - 4t \end{cases}$.
   Which point on this line is closest to P?

8. Find the shortest distance from the point $P(4, 1, 9)$ to the plane $x - 4z = 2$. Which point on this plane is closest to P?

9. Let $\overrightarrow{u} = \begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix}$, $\overrightarrow{v} = \begin{bmatrix} 1 \\ -2 \\ 3 \end{bmatrix}$, $\overrightarrow{w} = \begin{bmatrix} -3 \\ 1 \\ 2 \end{bmatrix}$ be vectors in $\mathbb{R}^3$. Compute:

   (a) $\overrightarrow{v} \times \overrightarrow{w}$ and then $\overrightarrow{u} \times (\overrightarrow{v} \times \overrightarrow{w})$;

   (b) $(\overrightarrow{u} \cdot \vec{w})\,\overrightarrow{v} - (\overrightarrow{u} \cdot \overrightarrow{v})\,\overrightarrow{w}$.

10. Find the areas of the sides of the parallelepiped determined by the vectors $\overrightarrow{AB}$, $\overrightarrow{AC}$, and $\overrightarrow{AD}$, where A, B, C and D are the points in Problem 4. What is the volume of this parallelepiped?

Recommended Problems:

Pages 165 - 167: 1 a, c; 3 a; 4 a; 5, 7 a; 9 a, b; 15, 20, 24
Pages 177 - 179: 1 a; 2 a, b; 3 a; 6, 8, 9, 10 a; 11 a; 12, 13, 14, 15, 16 a; 18, 19, 24 a
Page 185: 3 a; 4 a; 5 a.