MATHEMATICS 221 L05 FALL 2007
MIDTERM EXAMINATION
Friday, November 2, 2007
Duration: 50 minutes


| I agree that this paper may be placed at the front of the classroom for pick-up. |  |  |  |  |
| ---: | :---: | :---: | :---: | :---: |
| Please initial either YES |  | or | NO |  |

NO CALCULATORS ALLOWED
ANSWER ALL QUESTIONS
SHOW ALL WORK

## LAST NAME

FIRST NAME
[5] 1. Solve the system:

$$
\begin{gathered}
3 x-2 y+z-5 u=5 \\
-6 x+4 y+z+7 u=-4
\end{gathered}
$$

2. Let $A=\left[\begin{array}{rr}2 & -3 \\ -1 & 2\end{array}\right]$. Express $A^{-1}$ as a product of elementary matrices.
[5] 3. Let $A$ be a square matrix. Prove that if $A^{2}$ is invertible then $A$ is invertible.
3. Given that $A^{-1}=\left[\begin{array}{rrr}7 & 2 & -6 \\ -3 & -1 & 3 \\ 2 & 1 & -2\end{array}\right]$. Find $\operatorname{adj} A$.
[10] 5. Given that $A$ and $B$ are $3 \times 3$ matrices such that $\operatorname{det} A=-1$ and $\operatorname{det} B=2$.
(a) Find $\operatorname{det}\left(2\left(B^{-1}\right)^{T} A^{3}\right)$.
(b) Find $\operatorname{det}\left(\operatorname{adj} A-A^{-1}\right)$.
[5] 6. Let $A=\left[\begin{array}{lll}1 & x & x \\ x & 1 & x \\ x & x & 1\end{array}\right]$. Find all values of the number $x$ so that $A$ is not invertible.
4. Let $T: \mathbb{R}^{2} \rightarrow \mathbb{R}^{2}$ defined by $T\left(\left[\begin{array}{l}x \\ y\end{array}\right]\right)=\left[\begin{array}{c}0 \\ y^{2}\end{array}\right]$ for any real numbers $x$ and $y$. Show that $T$ is not a linear transformation.
