MATHEMATICS 211 L08 B24 FALL 2010
QUIZ 2 SOLUTION
Friday, October 15, 2010 at 11:00

1. Given that $\left(A\begin{bmatrix}1&-1\\0&1\end{bmatrix}\right)^{-1}=\begin{bmatrix}2&3\\1&1\end{bmatrix}$. Find A.

Solution: Taking inverses both sides we get:

$A\begin{bmatrix}1&-1\\0&1\end{bmatrix}=\begin{bmatrix}-1&3\\1&-2\end{bmatrix}$ and so, $A=\begin{bmatrix}-1&3\\1&-2\end{bmatrix}\begin{bmatrix}1&-1\\0&1\end{bmatrix}^{-1}=\begin{bmatrix}-1&3\\1&-2\end{bmatrix}\begin{bmatrix}1&1\\0&1\end{bmatrix}=\begin{bmatrix}-1&2\\1&-1\end{bmatrix}$

2. Show that the statement: "If $A^2=A$ and $A\neq 0$ then A is invertible." is FALSE.

Solution: Let $A=\begin{bmatrix}1&0\\0&0\end{bmatrix}$. Then it is easy to see that $A^2=A$ and $A\neq 0$, but A is not invertible.

3. Let $A=\begin{bmatrix}2&3&1\\-3&-4&-1\\5&7&1\end{bmatrix}$

(a) Find A^{-1}.

Solution:

$$\begin{bmatrix}2&3&1&1&0&0\\-3&-4&-1&0&1&0\\5&7&1&0&0&1\end{bmatrix}\begin{matrix}\\R_1-R_3\\R_2+R_3\end{matrix}\begin{bmatrix}-3&-4&0&1&0&-1\\2&3&0&0&1&1\\5&7&1&0&0&1\end{bmatrix}R_1+R_2$$

$$\begin{bmatrix}-1&-1&0&1&1&0\\2&3&0&0&1&1\\5&7&1&0&0&1\end{bmatrix}\begin{matrix}\\R_2+2R_1\\R_3+5R_1\end{matrix}\begin{bmatrix}-1&-1&0&1&1&0\\0&1&0&2&3&1\\0&2&1&5&5&1\end{bmatrix}\begin{matrix}R_1+R_2\\\\R_3-2R_2\end{matrix}$$

$$\begin{bmatrix}-1&0&0&3&4&1\\0&1&0&2&3&1\\0&0&1&1&-1&-1\end{bmatrix}(-1)\,R_1\begin{bmatrix}1&0&0&-3&-4&-1\\0&1&0&2&3&1\\0&0&1&1&-1&-1\end{bmatrix}$$

Thus, $A^{-1}=\begin{bmatrix}-3&-4&-1\\2&3&1\\1&-1&-1\end{bmatrix}$

(b) Find X if $AX=\begin{bmatrix}1\\1\\1\end{bmatrix}$

Solution:

$$X=A^{-1}\begin{bmatrix}1\\1\\1\end{bmatrix}=\begin{bmatrix}-3&-4&-1\\2&3&1\\1&-1&-1\end{bmatrix}\begin{bmatrix}1\\1\\1\end{bmatrix}=\begin{bmatrix}-8\\6\\-1\end{bmatrix}$$

MATHEMATICS 221 L08 B22/23 FALL 2010
QUIZ 2 Wednesday, October 13, 2010 at 15:00

1. Let $A = \begin{bmatrix} 7 & 2 & -6 \\ -3 & -1 & 3 \\ 2 & 1 & -2 \end{bmatrix}$. Find A^{-1}. Make sure that you show all the work.

Solution:

$$\left[\begin{array}{ccc|ccc} 7 & 2 & -6 & 1 & 0 & 0 \\ -3 & -1 & 3 & 0 & 1 & 0 \\ 2 & 1 & -2 & 0 & 0 & 1 \end{array}\right] \begin{array}{c} R_1+2R_2 \\ \\ R_3+R_2 \end{array} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 2 & 0 \\ -3 & -1 & 3 & 0 & 1 & 0 \\ -1 & 0 & 1 & 0 & 1 & 1 \end{array}\right] \begin{array}{c} \\ R_2+3R_1 \\ R_3+R_1 \end{array}$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 2 & 0 \\ 0 & -1 & 3 & 3 & 7 & 0 \\ 0 & 0 & 1 & 1 & 3 & 1 \end{array}\right] (-1)\,R_2 \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 2 & 0 \\ 0 & 1 & -3 & -3 & -7 & 0 \\ 0 & 0 & 1 & 1 & 3 & 1 \end{array}\right] R_2+3R_3$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 2 & 0 \\ 0 & 1 & 0 & 0 & 2 & 3 \\ 0 & 0 & 1 & 1 & 3 & 1 \end{array}\right]$$

Thus, $A^{-1} = \begin{bmatrix} 1 & 2 & 0 \\ 0 & 2 & 3 \\ 1 & 3 & 1 \end{bmatrix}$

2. Find the matrix A given that $\left(A^T - 2I\right)^{-1} = 2\begin{bmatrix} 1 & 1 \\ 2 & 3 \end{bmatrix}$.

Solution: Taking inverses both sides we get:

$A^T - 2I = \frac{1}{2}\begin{bmatrix} 3 & -1 \\ -2 & 1 \end{bmatrix}$ and so, $A^T = \frac{1}{2}\begin{bmatrix} 3 & -1 \\ -2 & 1 \end{bmatrix} + 2I = \frac{1}{2}\begin{bmatrix} 3 & -1 \\ -2 & 1 \end{bmatrix} + \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} = \frac{1}{2}\begin{bmatrix} 7 & -1 \\ -2 & 5 \end{bmatrix}$.

Now, $A = \left(A^T\right)^T = \frac{1}{2}\begin{bmatrix} 7 & -2 \\ -1 & 5 \end{bmatrix}$.

3. Let A be a square matrix. Show that the statement (a) is false and that the statement (b) is true.

(a) If $A^2 = A$ and $A \neq 0$ then A is invertible.

Solution: Let $A = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$. Then it is clear that $A^2 = A$ and $A \neq 0$ but A is not invertible.

(b) If $A^2 = 0$ then $I - A$ is invertible and $(I - A)^{-1} = I + A$.

Solution: Suppose that $A^2 = 0$. Then $(I - A)(I + A) = I + A - A - A^2 = I - A^2 = I + 0 = I$ and $(I + A)(I - A) = I - A + A - A^2 = I - A^2 = I - 0 = I$, which means $I - A$ is invertible and $(I - A)^{-1} = I + A$.