1. (a) F
   (b) T
   (c) T
   (d) F
   (e) F
   (f) T
   (g) F
   (h) T

2. (a) $\begin{bmatrix} 1 & 0 & 8 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$
   (b) 1
   (c) 12
   (d) $\frac{3\pi}{4}$ (or $-\frac{5\pi}{4}$)

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4. (a) $A = \left[\begin{array}{cccc|c} 1 & -1 & -5 & 6 & 1 \\ 2 & 0 & -4 & 8 & 6 \\ -2 & 1 & 7 & -10 & -4 \\ -1 & 1 & 5 & -6 & -1 \end{array}\right]$

   (b) $R = \left[\begin{array}{cccc|c} 1 & 0 & -2 & 4 & 3 \\ 0 & 1 & 3 & -2 & 2 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right]$, Rank($A$)=2;

   (c) Gen. Sol. to the system: $X = \begin{bmatrix} 3 \\ 2 \\ 0 \\ 0 \end{bmatrix} + t \begin{bmatrix} 2 \\ -3 \\ 1 \\ 0 \end{bmatrix} + s \begin{bmatrix} -4 \\ 2 \\ 0 \\ 1 \end{bmatrix}$,

   with $t, s \in \mathbb{R}$;

   Particular solution: $X_1 = \begin{bmatrix} 3 \\ 2 \\ 0 \\ 0 \end{bmatrix}$;

(d) Gen. Sol. to the ass. hom. system: $X_2 = t \begin{bmatrix} 2 \\ -3 \\ 1 \\ 0 \end{bmatrix} + s \begin{bmatrix} -4 \\ 2 \\ 0 \\ 1 \end{bmatrix}$, with $t, s \in \mathbb{R}$

Basic solutions: $X_3 = \begin{bmatrix} 2 \\ -3 \\ 1 \\ 0 \end{bmatrix}$ and $X_4 = \begin{bmatrix} -4 \\ 2 \\ 0 \\ 1 \end{bmatrix}$.

5. (a) $A^{-1} = \begin{bmatrix} -3 & -1 & 5 \\ 5 & 2 & -8 \\ -1 & 0 & 1 \end{bmatrix}$;

   (b) $x = 1$, $y = -2$ and $z = 2$.

6. (a) $P = P^1$ is a stochastic matrix with nonzero entries;

   (b) $\frac{5}{9}$;

   (c) $S = \frac{1}{2} \begin{bmatrix} 1 \\ 1 \end{bmatrix}$.

7. A has two eigenvalues $\lambda_1 = -3$ of multiplicity 1, and $\lambda_2 = \lambda_3 = 3$ of multiplicity **Two**. There are **Two** basic eigenvectors of A, say $\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$ and $\begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$, corresponding to the eigenvalue $\lambda_2 = \lambda_3 = 3$. Therefore, A is diagonalizable.

8. (a) $7 + 4i$;

   (b) $512(-1 + \sqrt{3}i)$.

9. (a) $16e^{\frac{4\pi}{3}i}$ or $16e^{-\frac{2\pi}{3}i}$;

   (b) The complex number $z = -8 - 8\sqrt{3}i$ has 4 fourth roots: $z_o = 2e^{\frac{\pi}{3}i} = 1 + \sqrt{3}i$, $z_1 = 2e^{\frac{5\pi}{6}i} = -\sqrt{3} + i$, $z_2 = 2e^{\frac{4\pi}{3}i} = -1 - \sqrt{3}i$ and $z_3 = 2e^{\frac{11\pi}{6}i} = 2e^{-\frac{\pi}{6}i} = \sqrt{3} - i$.

10. (a) A has basic eigenvectors $X_1 = \begin{bmatrix} 1 \\ 3 \end{bmatrix}$ and $X_2 = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$ corresponding to the eigenvalues $\lambda_1 = 3$ and $\lambda_2 = 2$, respectively; So, $P = [X_1 \ X_2] = \begin{bmatrix} 1 & 1 \\ 3 & 2 \end{bmatrix}$, with inverse $P^{-1} = \begin{bmatrix} -2 & 1 \\ 3 & -1 \end{bmatrix}$, diagonalizes A:

$$P^{-1}AP = \begin{bmatrix} -2 & 1 \\ 3 & -1 \end{bmatrix} \begin{bmatrix} 0 & 1 \\ -6 & 5 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 3 & 2 \end{bmatrix} = \begin{bmatrix} 3 & 0 \\ 0 & 2 \end{bmatrix};$$

(b) $x_k = 3^k$.

11. 5.

12. (a) $\overrightarrow{AB} \times \overrightarrow{AC} = \begin{bmatrix} 2 \\ -3 \\ 1 \end{bmatrix} \times \begin{bmatrix} 1 \\ -4 \\ 0 \end{bmatrix} = \begin{vmatrix} \vec{\imath} & \vec{\jmath} & \vec{k} \\ 2 & -3 & 1 \\ 1 & -4 & 0 \end{vmatrix} = 4\vec{\imath} + \vec{\jmath} - 5\vec{k} = \begin{bmatrix} 4 \\ 1 \\ -5 \end{bmatrix};$

(b) $\vec{n} = \overrightarrow{AB} \times \overrightarrow{AC}$ is a normal vector. The plane through the points A, B and C has equation $4(x-1) + (y-2) - 5(z-2) = 0$ or $4x + y - 5z + 4 = 0$. It does not contain D as $4(1) + 3 - 5(2) + 4 = 1 \neq 0$;

(c) The triangle has area $\frac{1}{2}\|\overrightarrow{AB} \times \overrightarrow{AC}\| = \frac{1}{2}\sqrt{42}$;

(d) 4.