THE UNIVERSITY OF CALGARY
DEPARTMENT OF MATHEMATICS AND STATISTICS
FINAL EXAMINATION
MATHEMATICS 211 L06 - Fall, 2008

Time: Three hours

I.D. NUMBER	SURNAME	OTHER NAMES

STUDENT IDENTIFICATION

Each candidate must sign the Seating List confirming presence at the examination. All candidates for final examinations are required to place their University of Calgary student I.D. cards on their desks for the duration of the examination. (Students writing mid-term tests can also be asked to provide identity proof.) Students without an I.D. card who can produce an **acceptable** alternative I.D., e.g., one with a printed name and photograph, are allowed to write the examination.

A student without acceptable I.D. will be required to complete an Identification Form. The form indicates that there is no guarantee that the examination paper will be graded if any discrepancies in identification are discovered after verification with the student's file. **A student who refuses to produce identification or who refuses to complete and sign the Identification Form is not permitted to write the examination.**

EXAMINATION RULES

1. Students late in arriving will not normally be admitted after one-half hour of the examination time has passed.
2. No candidate will be permitted to leave the examination room until one-half hour has elapsed after the opening of the examination, nor during the last 15 minutes of the examination. All candidates remaining during the last 15 minutes of the examination period must remain at their desks until their papers have been collected by an invigilator.
3. All enquiries and requests must be addressed to supervisors only.
4. **Candidates are strictly cautioned against**:
   (a) speaking to other candidates or communicating with them under any circumstances whatsoever;
   (b) bringing into the examination room any textbook, notebook or memoranda not authorized by the examiner;
   (c) making use of calculators and/or portable computing machines not authorized by the instructor;
   (d) leaving answer papers exposed to view;
   (e) attempting to read other students' examination papers.

   The penalty for violation of these rules is suspension or expulsion or such other penalty as may be determined.
5. Candidates are requested to write on both sides of the page, unless the examiner has asked that the left half page be reserved for rough drafts or calculations.
6. Discarded matter is to be struck out and not removed by mutilation of the examination answer book.
7. Candidates are cautioned against writing in their answer book any matter extraneous to the actual answering of the question set.
8. The candidate is to write his/her name on each answer book as directed and is to number each book.
9. During the examination a candidate must report to a supervisor before leaving the examination room.
10. Candidates must stop writing when the signal is given. Answer books must be handed to the supervisor-in-charge promptly. Failure to comply with these regulations will be cause for rejection of an answer paper.
11. If during the course of an examination a student becomes ill or receives word of domestic affliction, the student must report at once to the supervisor, hand in the unfinished paper and request that it be cancelled. If physical and/or emotional ill health is the cause, the student must report at once to a physician/counsellor so that subsequent application for a deferred examination is supported by a completed Physical/Counsellor Statement form. Students can consult professionals at University Health Services or Counselling and Student Development Centre during normal working hours or consult their physician/counsellor in the community. **Once an examination has been handed in for marking a student cannot request that the examination be cancelled for whatever reason. Such a request will be denied. Retroactive withdrawals will also not be considered.**

Question	Total Marks	Actual Marks
1	8	
2	6	
3	6	
4	28	
5	11	
6	15	
7	12	
8	10	
9	18	
10	24	
11	8	
12	14	
Total	160	

NOTE: No formula sheets or calculators are allowed.

1. In each answer space below, enter T if the statement is always true, enter F if the statement is not always true: Let A be a square matrix of size $n \times n$ and B an arbitrary n-column. [8]

____**(a)** If A has rank n, then 0 is an eigenvalue of A.

____**(b)** If $AX = 0$ for some nonzero n-column X, then $\det(A) = 0$.

____**(c)** If A is a product of elementary matrices, then $AX = B$ has a unique solution.

____**(d)** If $AX = 0$ has infinitely many solutions, then $AX = B$ also has infinitely many solutions.

____**(e)** If $AX = B$ has no solutions, then $AX = 0$ has only the trivial solution $X = 0$.

____**(f)** If $AX = B$ can be solved by Cramer's rule, then it is possible to bring A to the identity matrix by row operations.

____**(g)** If just a single row operation transforms A into a row-echelon form, then A is an elementary matrix.

____**(h)** A and A^T have the same determinant as well as the same eigenvalues.

2. Quick Questions, no explanation needed. Enter your quick answer in the space provided.

(a) The inverse of the matrix $A = \begin{bmatrix} 1 & 0 & -8 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ is $A^{-1} = \begin{bmatrix} \_ & \_ & \_ \\ \_ & \_ & \_ \\ \_ & \_ & \_ \end{bmatrix}$. [1]

(b) Let $A = \begin{bmatrix} 2 & 1 & -3 \\ 3 & -2 & 1 \\ -1 & 3 & 2 \end{bmatrix}$, $B = \begin{bmatrix} 1 \\ -2 \\ 3 \end{bmatrix}$ and $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$. If $AX = B$ and $\det(A) = -42$, then the value of y is: ____. [2]

(c) The matrix $A = \begin{bmatrix} 3 & 0 & 4 & 96 \\ 0 & 0 & 1 & 0 \\ 9 & -2 & 12 & 23 \\ 0 & 0 & 15 & 2 \end{bmatrix}$ has determinant $\det(A) =$____. [2]

(d) The angle $\theta =$____ is an argument of the complex number $z = -3e^{-\frac{\pi}{4}i}$. [1]

3. Prove that the midpoints of any quadrilateral are vertices of a parallelogram. [6]

4. Consider the following system of linear equations:

$$\begin{cases} x_1 - x_2 - 5x_3 + 6x_4 &= 1 \\ 2x_1 \quad - 4x_3 + 8x_4 &= 6 \\ -2x_1 + x_2 + 7x_3 - 10x_4 &= -4 \\ -x_1 + x_2 + 5x_3 - 6x_4 &= -1. \end{cases}$$

(a) Write down the augmented matrix A for this system. [2]

(b) Find the reduced row-echelon form R of A. What is the rank of A? [10]

Problem 4. continued.

(c) Use the reduced row-echelon form R of A (from part (a)) to find the general solution to the system. Give a particular solution. [10]

(d) Give the general solution of the associated homogeneous system and specify basic solutions. [6]

5. (a) Find the inverse A^{-1} of the 3×3 matrix $A = \begin{bmatrix} 2 & 1 & -2 \\ 3 & 2 & 1 \\ 2 & 1 & -1 \end{bmatrix}$. [8]

(b) Use the inverse A^{-1} of A from part (a) to solve [3]

$$\begin{cases} 2x + y - 2z = -4 \\ 3x + 2y + z = 1 \\ 2x + y - z = -2. \end{cases}$$

6. Consider a Markov chain that starts in state 1 with transition matrix $P = \begin{bmatrix} \frac{1}{3} & \frac{2}{3} \\ \frac{2}{3} & \frac{1}{3} \end{bmatrix}$.

   (a) Explain why this chain is regular. [4]

   (b) Find the probability for the chain to be in state 1 after 2 transitions. [5]

Problem 6. continued.

(c) Find the steady-state vector S for the chain. [6]

[12]

7. Determine whether the matrix $A = \begin{bmatrix} 1 & 0 & 2 \\ 2 & 3 & -2 \\ 4 & 0 & -1 \end{bmatrix}$ is diagonalizable or not. Explain your answer.

α	$\pi/6$	$\pi/4$	$\pi/3$	$\pi/2$	$2\pi/3$	$3\pi/4$	$5\pi/6$	$4\pi/3$	$11\pi/6$
$\cos\alpha$	$\sqrt{3}/2$	$\sqrt{2}/2$	$1/2$	0	$-1/2$	$-\sqrt{2}/2$	$-\sqrt{3}/2$	$-1/2$	$\sqrt{3}/2$
$\sin\alpha$	$1/2$	$\sqrt{2}/2$	$\sqrt{3}/2$	1	$\sqrt{3}/2$	$\sqrt{2}/2$	$1/2$	$-\sqrt{3}/2$	$-1/2$

8. Express the following complex numbers in the form $a + b\,i$:

(a) $z = \dfrac{26 - 13i}{2 - 3i}$ [4]

(b) $z = (-1 + \sqrt{3}\,i)^{10}$. [6]

9. (a) Express the complex number $z = -8 - 8\sqrt{3}\,i$ in polar form. [6]

(b) Express the 4 fourth roots of the complex number $z = -8 - \sqrt{3}\,i$ in polar form and in the form $a + b\,i$. [12]

10. (a) Diagonalize the 2×2 matrix $A = \begin{bmatrix} 0 & 1 \\ -6 & 5 \end{bmatrix}$. [12]

Problem 10. continued.

(b) Use the diagonalization of the matrix A from part (a) to solve the linear recurrence $x_{k+2} = -6x_k + 5x_{k+1}$ with $x_0 = 1$ and $x_1 = 3$, i.e., find the general term x_k of the recursive sequence. [12]

11. Find the shortest distance from the point $P(2, -5, -2)$ to the line $(l) \equiv \begin{cases} x = 4 + 2t \\ y = -4 - 3t \\ z = 5 + 4t \end{cases}$. [8]

12. Let $A(1, 2, 1)$, $B(3, -1, 2)$, $C(2, -2, 1)$ and $D(1, 3, 2)$ be four points in the 3-dimensional space.

    (a) Find the cross product of the vectors $\overrightarrow{AB}$ and $\overrightarrow{AC}$. [3]

    (b) Find the equation of the plane Π passing through the points A, B and C. Determine whether the plane Π contains the point D. [4]

Problem 12. continued.

(c) Find the area of the triangle with A, B and C as vertices. [3]

(d) Find the volume of the parallelepiped determined by the vectors $\overrightarrow{AB}$, $\overrightarrow{AC}$ and $\overrightarrow{AD}$. [4]

End of Examination