1. (a) $A = \begin{bmatrix} 5 & 3 \\ 1 & 1 \end{bmatrix}$;

   (b) $A_T = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$. (Counterclockwise rotation)

2. (a) $K = [A|B] = \left[\begin{array}{cccc|c} -2 & 3 & -6 & -1 & 11 \\ 1 & -1 & 2 & 2 & -4 \\ 1 & -2 & 4 & -1 & -7 \end{array}\right]$

   (b) $R = \left[\begin{array}{cccc|c} 1 & 0 & 0 & 5 & -1 \\ 0 & 1 & -2 & 3 & 3 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right]$; $U = \begin{bmatrix} 1 & 3 & 0 \\ 1 & 2 & 0 \\ 1 & 1 & 1 \end{bmatrix}$, ($U$ is not unique, e.g., $U = \begin{bmatrix} 0 & 2 & -1 \\ 0 & 1 & -1 \\ 1 & 1 & 1 \end{bmatrix}$, or a different one).

   (c) Gen. sol. to the syst.: $X = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} -1 \\ 3 \\ 0 \\ 0 \end{bmatrix} + t \begin{bmatrix} 0 \\ 2 \\ 1 \\ 0 \end{bmatrix} + s \begin{bmatrix} -5 \\ -3 \\ 0 \\ 1 \end{bmatrix}$;

   (d) Particular solution: $\begin{bmatrix} -1 \\ 3 \\ 0 \\ 0 \end{bmatrix}$;

   (e) Gen. sol. to the associated homog. syst.: $X_0 = t \begin{bmatrix} 0 \\ 2 \\ 1 \\ 0 \end{bmatrix} + s \begin{bmatrix} -5 \\ -3 \\ 0 \\ 1 \end{bmatrix}$; Basic solutions: $\begin{bmatrix} 0 \\ 2 \\ 1 \\ 0 \end{bmatrix}$ and $\begin{bmatrix} -5 \\ -3 \\ 0 \\ 1 \end{bmatrix}$.

3. (a) $h = -4$;

(b) $h = -20$.

4. $A^{-1} = \begin{bmatrix} -3 & 3 & -17 \\ 0 & 1 & -2 \\ -1 & 0 & -4 \end{bmatrix}$.

5. $A = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$. (This product is not unique.)

6. (a) $a = 7x - 5y$ and $b = -4x + 3y$;

(b) $T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = (7x - 5y)\begin{bmatrix} 1 \\ 1 \end{bmatrix} + (-4x + 3y)\begin{bmatrix} 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 3x - 2y \\ -x + y \end{bmatrix}$;

(c) $T\left(\begin{bmatrix} 1 \\ 0 \end{bmatrix}\right) = \begin{bmatrix} 3 \\ -1 \end{bmatrix}$ and $T\left(\begin{bmatrix} 0 \\ 1 \end{bmatrix}\right) = \begin{bmatrix} -2 \\ 1 \end{bmatrix}$;

(d) $A_T = \begin{bmatrix} 3 & -2 \\ -1 & 1 \end{bmatrix}$;

(e) Yes. $A_{T^{-1}} = \begin{bmatrix} 1 & 2 \\ 1 & 3 \end{bmatrix}$

7. (a) $S_0 = [1\ 0]^T$, $S_2 = PS_0$, $S_2 = PS_1 = PPS_0 = P^2S_0 = [11/16\ \mathbf{5/16}]^T$. Therefore, the probability to be in state 2 after 2 transitions is 5/16.

(b) Since $P^1 = P$ has no zero entries, the chain is regular. Its steady state-vector S is the only probability vector which solves the homogeneous system $(I_2 - P)X = 0$. $S = [2/3\ 1/3]^T$.