UNIVERSITY OF CALGARY
DEPARTMENT OF MATHEMATICS AND STATISTICS

MATHEMATICS 211 — L01 Fall 2009

MIDTERM EXAM A [November 05, 2009 (Thursday)]

Time: 75 minutes. PLEASE write your Name on the very last page.
NO CALCULATORS.
Total Marks = 100. Work all problems. Marks are shown in brackets. Write your answers in the space provided.

Student ID: ____________________

[Marks]

[6] 1. (a) Solve the following matrix equation for A: $(\frac{1}{2}A^T)^{-1} = \begin{bmatrix} 1 & -1 \\ -3 & 5 \end{bmatrix}$.

Then

$$A = \begin{bmatrix} \_\_\_ & \_\_\_ \\ \_\_\_ & \_\_\_ \end{bmatrix};$$

[6] (b) Consider the transformation T of vectors of $\mathbb{R}^2$ defined as follows: Rotation through $\frac{\pi}{2}$ followed by reflection in the y-axis. Then the matrix A_T of T is

$$A_T = \begin{bmatrix} \_\_\_ & \_\_\_ \\ \_\_\_ & \_\_\_ \end{bmatrix}.$$

2. Let $\begin{cases} -2x_1 + 3x_2 - 6x_3 - x_4 &= 11 \\ x_1 - x_2 + 2x_3 + 2x_4 &= -4 \\ x_1 - 2x_2 + 4x_3 - x_4 &= -7 \end{cases}$.

[2] (a) The system has augmented matrix $K = \left[\begin{array}{cccc|c} \_\_\_ & \_\_\_ & \_\_\_ & \_\_\_ & \_\_\_ \\ \_\_\_ & \_\_\_ & \_\_\_ & \_\_\_ & \_\_\_ \\ \_\_\_ & \_\_\_ & \_\_\_ & \_\_\_ & \_\_\_ \end{array}\right]$;

[14] (b) Find the reduced row-echelon form R of K and an invertible matrix U of size 3×3 such that $U\,K = R$:

$$R = \left[\begin{array}{cccc|c} \_\_\_ & \_\_\_ & \_\_\_ & \_\_\_ & \_\_\_ \\ \_\_\_ & \_\_\_ & \_\_\_ & \_\_\_ & \_\_\_ \\ \_\_\_ & \_\_\_ & \_\_\_ & \_\_\_ & \_\_\_ \end{array}\right] \quad \text{and} \quad U = \left[\begin{array}{ccc} \_\_\_ & \_\_\_ & \_\_\_ \\ \_\_\_ & \_\_\_ & \_\_\_ \\ \_\_\_ & \_\_\_ & \_\_\_ \end{array}\right]$$

Problem 2. continued.

[6] (c) Use the reduced row-echelon form R of K from part (b) to find the general solution to this system:

$$X = \begin{bmatrix} \_\_\_ \\ \_\_\_ \\ \_\_\_ \\ \_\_\_ \end{bmatrix} + t \begin{bmatrix} \_\_\_ \\ \_\_\_ \\ \_\_\_ \\ \_\_\_ \end{bmatrix} + s \begin{bmatrix} \_\_\_ \\ \_\_\_ \\ \_\_\_ \\ \_\_\_ \end{bmatrix};$$

[2] (d) $X_1 = \begin{bmatrix} \_\_\_ \\ \_\_\_ \\ \_\_\_ \\ \_\_\_ \end{bmatrix}$ is a particular solution to the system;

(e)

$$X_2 = \begin{bmatrix} \_\_\_ \\ \_\_\_ \\ \_\_\_ \\ \_\_\_ \end{bmatrix} \quad \text{and} \quad X_3 = \begin{bmatrix} \_\_\_ \\ \_\_\_ \\ \_\_\_ \\ \_\_\_ \end{bmatrix}$$

[4] are basic solutions to the associated homogeneous system.

3. (a) Determine the value of h for which the matrix below is the augmented matrix of a consistent system.

$$\left[\begin{array}{cc|c} 6 & -8 & h \\ -12 & 16 & 8 \end{array}\right]$$

[4] $h =$ ____;

(b) Determine the value of h for which the matrix below is the augmented matrix of a linear system with infinitely many solutions.

$$\left[\begin{array}{cc|c} 6 & -5 & 8 \\ 24 & h & 32 \end{array}\right]$$

[4] $h =$ ____.

[16] 4. Use row operations to find the inverse A^{-1} of $A = \begin{bmatrix} -4 & 12 & 11 \\ 2 & -5 & -6 \\ 1 & -3 & -3 \end{bmatrix}$.

Then

$$A^{-1} = \begin{bmatrix} \_\_\_ & \_\_\_ & \_\_\_ \\ \_\_\_ & \_\_\_ & \_\_\_ \\ \_\_\_ & \_\_\_ & \_\_\_ \end{bmatrix}$$

[10] 5. Express the matrix $A = \begin{bmatrix} 2 & 5 \\ 1 & 2 \end{bmatrix}$ as a product of 3 elementary matrices.

$$A = \begin{bmatrix} 2 & 5 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} \_\_\_ & \_\_\_ \\ \_\_\_ & \_\_\_ \end{bmatrix} \begin{bmatrix} \_\_\_ & \_\_\_ \\ \_\_\_ & \_\_\_ \end{bmatrix} \begin{bmatrix} \_\_\_ & \_\_\_ \\ \_\_\_ & \_\_\_ \end{bmatrix}.$$

6. Let $T\colon \mathbb{R}^2 \longrightarrow \mathbb{R}^2$ be a linear transformation of vectors in $\mathbb{R}^2$ such that $T\left(\begin{bmatrix} 3 \\ 4 \end{bmatrix}\right) = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ and $T\left(\begin{bmatrix} 5 \\ 7 \end{bmatrix}\right) = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$.

(a) Express the vector $\begin{bmatrix} x \\ y \end{bmatrix}$ as a linear combination of $\begin{bmatrix} 3 \\ 4 \end{bmatrix}$ and $\begin{bmatrix} 5 \\ 7 \end{bmatrix}$:

$$\begin{bmatrix} x \\ y \end{bmatrix} = a \begin{bmatrix} 3 \\ 4 \end{bmatrix} + b \begin{bmatrix} 5 \\ 7 \end{bmatrix},$$

[4] where $a =$ __________ and $b =$ __________. (a and b depend on x and y)

[4] (b) Find a formula for $T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right)$:

$$T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \underline{\qquad\qquad} \begin{bmatrix} \underline{\quad} \\ \underline{\quad} \end{bmatrix} + \underline{\qquad\qquad} \begin{bmatrix} \underline{\quad} \\ \underline{\quad} \end{bmatrix} = \begin{bmatrix} \underline{\qquad\qquad} \\ \underline{\qquad\qquad} \end{bmatrix};$$

[2] (c) Use the formula from part (b) to find

$$T\left(\begin{bmatrix} 1 \\ 0 \end{bmatrix}\right) = \begin{bmatrix} \underline{\quad} \\ \underline{\quad} \end{bmatrix} \quad \text{and} \quad T\left(\begin{bmatrix} 0 \\ 1 \end{bmatrix}\right) = \begin{bmatrix} \underline{\quad} \\ \underline{\quad} \end{bmatrix};$$

[2] (d) The matrix A_T of T is $A_T = \begin{bmatrix} \underline{\quad} & \underline{\quad} \\ \underline{\quad} & \underline{\quad} \end{bmatrix}$;

[2] (e) Is T invertible? Yes or no: ____. If Yes, then its inverse transformation T^{-1} is induced by the matrix $A_{T^{-1}} = \begin{bmatrix} \underline{\quad} & \underline{\quad} \\ \underline{\quad} & \underline{\quad} \end{bmatrix}$.

7. Let $P = \begin{bmatrix} 3/4 & 1/2 \\ 1/4 & 1/2 \end{bmatrix}$ be the transition matrix of a Markov chain that starts in state 1.

[4] (a) The probability for the chain to be in state 2 after 2 transitions is ______;

(b) The steady-state vector for this chain is

$$S = \begin{bmatrix} \_\_\_\_ \\ \\ \_\_\_\_ \end{bmatrix}.$$

[8]

Name:	Student ID:	Marks: