The University of Calgary
Department of Mathematics and Statistics
Math 211 L60 Summer 2009 Wednesday Tut (June 10, 2009)
Quiz # 3 Duration: 50 minutes
[marks] Total marks = 30

Name:_____I.D.#:_____

1. **[4]** The reflection of vectors in $\mathbb{R}^2$ in the line $y = x$ followed by the The rotation (counterclockwise) about the origin through the angle $\pi/2$, is a linear transformation. Find the corresponding matrix.

2. Let $T\colon \mathbb{R}^2 \longrightarrow \mathbb{R}^2$ be a linear transformation of vectors in $\mathbb{R}^2$ such that $T\Big(\begin{bmatrix} 2 \\ 1 \end{bmatrix}\Big) = \begin{bmatrix} 1 \\ 3 \end{bmatrix}$ and $T\Big(\begin{bmatrix} 3 \\ 2 \end{bmatrix}\Big) = \begin{bmatrix} 1 \\ 4 \end{bmatrix}$.

   (a) **[4]** Find a formula for $T\Big(\begin{bmatrix} x \\ y \end{bmatrix}\Big)$; (Hint: Express $\begin{bmatrix} x \\ y \end{bmatrix}$ as a linear combination of $\begin{bmatrix} 2 \\ 1 \end{bmatrix}$ and $\begin{bmatrix} 3 \\ 2 \end{bmatrix}$)

Problem 2. continued.

(b) **[2]** Use the formula from part (a) to find $T\Big(\begin{bmatrix} 1 \\ 0 \end{bmatrix}\Big)$ and $T\Big(\begin{bmatrix} 0 \\ 1 \end{bmatrix}\Big)$;

(c)**[2]** Find the matrix of T;

(d) **[2]** Is T invertible? If yes, find the matrix of the inverse transformation T^{-1}.

3. [**4**] Express the following matrix as a product of elementary martices:

$$A = \begin{bmatrix} 9 & 7 \\ 4 & 3 \end{bmatrix}.$$

4. **[6]** Given $A = \begin{bmatrix} 2 & 4 & 1 & 4 \\ 1 & 2 & 1 & 1 \\ 1 & 2 & 0 & 3 \end{bmatrix}$, find an invertible matrix U such that the product UA is the reduced row-echelon form of A.

5. Consider a Markov chain that starts in state 1 with transition matrix $P = \begin{bmatrix} 0 & 2/3 & 1/3 \\ 1/3 & 0 & 2/3 \\ 2/3 & 1/3 & 0 \end{bmatrix}$.

(a) **[3]** Show that this chain is regular. What is the probalibility that the chain is in state 2 after 2 transitions?

(b) **[3]** Find the steady-state vector for the chain.