Practice Problems S2

1. Express the vector $X = \begin{bmatrix} -1 \\ 2 \end{bmatrix}$ as a linear combination of vectors $X_1 = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$ and $X_2 = \begin{bmatrix} 3 \\ 4 \end{bmatrix}$.

2. Show that the vectors $X_1 = \begin{bmatrix} 2 \\ -1 \\ 3 \end{bmatrix}$, $X_2 = \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}$ and $X_3 = \begin{bmatrix} -1 \\ 1 \\ 1 \end{bmatrix}$ are linearly independent.

3. Consider the following system of linear equations:

$$\begin{cases} x_1 - 2x_2 + x_3 - 4x_4 = 1 \\ x_1 + 3x_2 + 7x_3 + 2x_4 = 2 \\ x_1 - 12x_2 - 11x_3 - 16x_4 = -1 \end{cases}.$$

   (a) Find basic solutions to the associated homogeneous system;
   (b) Find a particular solution to the system.

4. Find the general solution to the linear system $AX = B$ and specify a particular solution, where

$$A = \begin{bmatrix} 2 & 1 & -1 & -1 \\ 3 & 1 & 1 & -2 \\ -1 & -1 & 2 & 1 \\ -2 & -1 & 0 & 2 \end{bmatrix} \quad \text{and} \quad B = \begin{bmatrix} -1 \\ -2 \\ 2 \\ 3 \end{bmatrix}.$$

   Find basic solutions and write the general solution to the associated homogeneous system as a linear combination of these basic solutions.

5. Let $A = \begin{bmatrix} 1 & -1 & 2 \\ 2 & 0 & 4 \end{bmatrix}$, $B = \begin{bmatrix} 2 & 3 & 1 \\ 1 & 9 & 7 \\ -1 & 0 & 2 \end{bmatrix}$ and $C = \begin{bmatrix} 1 & 3 & 2 \\ 1 & 1 & 1 \\ -1 & 4 & 1 \end{bmatrix}$.

   Compute the products AB, BA^T, BC and CB.

6. Consider a directed graph with three vertices v_1, v_2 and v_3. Find the adjacency matrix of this graph if the edges are $v_1 \longrightarrow v_1$, $v_1 \longrightarrow v_2$, $v_2 \longrightarrow v_3$, $v_3 \longrightarrow v_2$ and $v_3 \longrightarrow v_1$. Determine the number of paths of length 5 from v_2 to v_3 and from v_3 to v_1.