

Practice Problems S3

1. Show that the matrices $A = \begin{bmatrix} 3 & 4 \\ 4 & 5 \end{bmatrix}$ and $B = \begin{bmatrix} -5 & 4 \\ 4 & -3 \end{bmatrix}$ are inverses of each other.

2. Find the inverses of the following matrices:

(a) $\begin{bmatrix} 7 & 4 \\ 3 & 2 \end{bmatrix}$; (b) $\begin{bmatrix} 1 & 3 & 2 \\ 1 & 1 & 1 \\ -1 & 4 & 1 \end{bmatrix}$.

3. Use matrix inversion to solve the following systems of linear equations:

(a) $\begin{cases} 7x + 4y = 2 \\ 3x + 2y = -2 \end{cases}$; (b) $\begin{cases} x + 3y + 2z = 5 \\ x + y + z = 1 \\ -x + 4y + z = 5 \end{cases}$.

4. Find A if

(a) $\left(A \begin{bmatrix} 1 & -1 \\ 1 & 0 \end{bmatrix} \right)^{-1} = \begin{bmatrix} 3 & 2 \\ 1 & 1 \end{bmatrix}$;

(b) $(A^{-1} - 2I_2)^T = -2 \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}$.

5. Determine whether the following matrices are elementary matrices or not; write down the inverses of the elementary matrices (explain your answer):

(a) $\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$, (b) $\begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix}$, (c) $\begin{bmatrix} 1 & 2 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$, (d) $\begin{bmatrix} 1 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$,

(e) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix}$, (f) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 2 \\ 0 & 0 & -1 \end{bmatrix}$.

6. Find an invertible matrix U such that the product $R = UA$ is the reduced row-echelon form of A if

$$A = \begin{bmatrix} 1 & -1 & 3 & 5 \\ 3 & -2 & 1 & -2 \\ -1 & 1 & 1 & 3 \end{bmatrix}.$$

7. Express the following matrix as a product of elementary matrices:

$$A = \begin{bmatrix} 5 & 3 \\ 2 & 1 \end{bmatrix}.$$

8. Find the matrix of the reflection in the line $y = -x$.
9. Find a rotation or a reflection that is equal to
- (a) reflection in the y -axis followed by rotation through $\pi/2$;
 - (b) rotation through $\pi/2$ followed by reflection in the line $y = x$.
10. Given $T([1 \ -2]^T) = [3 \ 4]^T$ and $T([-2 \ 5]^T) = [-1 \ 4]^T$, find $T([-4 \ 3]^T)$ if T is a linear transformation.