Practice Problems S4

1. Consider a Markov chain that starts in state 1 with transition matrix $P = \begin{bmatrix} \frac{1}{3} & \frac{2}{3} \\ \frac{2}{3} & \frac{1}{3} \end{bmatrix}$.

   (a) Explain why the chain is regular.

   (b) Find the probability that the chain is in state 1 after 2 transitions.

   (c) Find the steady-state vector for the chain.

2. By inspection, find the determinants of the following matrices:
   (a) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$; (b) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & -3 & 0 \\ 0 & 0 & 1 \end{bmatrix}$; (c) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 3 & 0 & 1 \end{bmatrix}$;
   (d) $\begin{bmatrix} 1 & -2 & 3 \\ 2 & 1 & 4 \\ 2 & -4 & 6 \end{bmatrix}$; (e) $\begin{bmatrix} 1 & 0 & 4 & 9 \\ -8 & -7 & 12 & 2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 5 & 3 \end{bmatrix}$.

3. Compute the determinants of the following matrices
   (a) $A = \begin{bmatrix} -2 & 1 & 3 \\ 1 & -7 & 4 \\ -2 & 1 & 3 \end{bmatrix}$; (b) $A = \begin{bmatrix} 3 & 5 & -2 & 6 \\ 1 & 2 & -1 & 1 \\ 2 & 4 & 1 & 5 \\ 3 & 7 & 5 & 3 \end{bmatrix}$.

4. Find the inverse of $A = \begin{bmatrix} -1 & 0 & 1 \\ 1 & 1 & 0 \\ -1 & -1 & 1 \end{bmatrix}$ using the adjoint formula.

5. Given $A = \begin{bmatrix} 3 & -1 & 2 \\ 5 & 5 & -2 \\ 1 & 2 & 3 \end{bmatrix}$, find the $(1,3)$-entry of A^{-1}.

6. If A and B are 4×4-matrices with $\det(A) = -2$ and $\det(B) = 2$, find:

   (a) $\det\left(\text{adj}(A)B^T A^4 (B^2)^T\right)$; (b) $\det\left(A^3 (B^2)^T ((\text{adj}(A))^{-1})^3 B^{-1}\right)$.

7. Let $A = \begin{bmatrix} a & b & c \\ 1 & -1 & 2 \\ d & e & f \end{bmatrix}$, $A = \begin{bmatrix} a & b & c \\ 3 & -2 & 1 \\ d & e & f \end{bmatrix}$ and $C = \begin{bmatrix} a & b & c \\ 1 & 0 & -3 \\ d & e & f \end{bmatrix}$ be 3×3-matrices. If $\det(A) = 4$ and $\det(B) = 5$, find $\det(C)$.

8. For which values of $c \in \mathbb{R}$ is A invertible if $A = \begin{bmatrix} 1 & c & 0 \\ 2 & 0 & c \\ c & -1 & 1 \end{bmatrix}$.

9. Solve the following system by Cramer's rule:

   (a) $\begin{cases} x + 2y = 4 \\ 3x + 7y = 13 \end{cases}$; (b) $\begin{cases} 3x - 2y + 4z = -3 \\ 5x + 3y + z = 0 \\ 2x + 6y - 5z = 6 \end{cases}$.

Recommended Problems:

Pages 133-134: 1. a, b, f, g, h, k, l, m, n, o, p; 5. a, b; 6, 7, 8, 9, 10, 13, 14a, 15a;
Pages 145-146: 1. a, c; 2. a, b, c, d; 3, 4, 5, 6b, 8, 9, 10.