UNIVERSITY OF CALGARY
DEPARTMENT OF MATHEMATICS AND STATISTICS

MATHEMATICS 211 — L06 Fall 2008

MIDTERM EXAM [October 31, 2008 (Monday)]

Time: 50 minutes. PLEASE write your Name on the very last page.
NO CALCULATORS.
Total Marks = 100. Work all problems. Marks are shown in brackets.

Student ID: ____________________

[Marks]

1. Let A be the 3×4 matrix:

$$A = \begin{bmatrix} 3 & 2 & -7 & 11 & 2 \\ 2 & 1 & -4 & 7 & 2 \\ 1 & 1 & -3 & 4 & 0 \end{bmatrix}.$$

[12] (a) Find an invertible matrix U of size 3×3 such that the product $U\,A = R$ is the reduced row-echelon form of A.

Problem 1. continued.

[2] (b) Write down the system of linear equations whose *augmented* matrix is A.

(c) Use the reduced row-echelon matrix R of A from part (a) to find the **general solution to this system** (see part (b)) and specify a **particular** solution to the system. What is the
[12] **general solution to the associated homogeneous system**? Find **basic solutions**.

[6] 2. (a) Use matrix inversion to find x and y if

$$\begin{cases} 2x + 5y = 1 \\ 3x + 4y = -2. \end{cases}$$

(b) Find $T\left(\begin{bmatrix} 1 \\ -2 \end{bmatrix}\right)$ if $T : \mathbb{R}^2 \longrightarrow \mathbb{R}^2$ is a linear transformation with $T\left(\begin{bmatrix} 2 \\ 3 \end{bmatrix}\right) = \begin{bmatrix} 2 \\ -5 \end{bmatrix}$

[8] and $T\left(\begin{bmatrix} 5 \\ 4 \end{bmatrix}\right) = \begin{bmatrix} 3 \\ 7 \end{bmatrix}$.

[18] 3. Use row operations to find the inverse of $A = \begin{bmatrix} 4 & -2 & 3 \\ 1 & 2 & -5 \\ 2 & 1 & -3 \end{bmatrix}$.

4. Let $A = \begin{bmatrix} 1 & -2 & 3 \\ 1 & 2 & -5 \\ 2 & 1 & -3 \end{bmatrix}$.

[8] (a) Compute the determinant of A;

[6] (b) Find the $(2, 3)$-entry of the inverse A^{-1} of A.

[12] 5. Express the matrix $A = \begin{bmatrix} 3 & 7 \\ 1 & 2 \end{bmatrix}$ as a product of elementary matrices.

6. Let $P = \begin{bmatrix} 3/5 & 1/3 \\ 2/5 & 2/3 \end{bmatrix}$ be the transition matrix of a Markov chain that starts in state 1.

[6] (a) What is the probability that the chain is in state 2 after 2 transitions?

[10] (b) Explain why this chain is regular and find the steady-state vector for the chain.

Name:	Student ID:	Marks: