MATHEMATICS 211 L03 WINTER 2010
QUIZ 2 SOLUTION
Wednesday, February 10, 2010 at 9:00

1. Let $A = \begin{bmatrix} 7 & 2 & -6 \\ -3 & -1 & 3 \\ 2 & 1 & -2 \end{bmatrix}$. Find A^{-1}. Make sure that you show all the work.

Solution:

$$\left[\begin{array}{ccc|ccc} 7 & 2 & -6 & 1 & 0 & 0 \\ -3 & -1 & 3 & 0 & 1 & 0 \\ 2 & 1 & -2 & 0 & 0 & 1 \end{array}\right] \begin{array}{l} \\ R_1 + 2R_2 \\ \\ \end{array} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 2 & 0 \\ -3 & -1 & 3 & 0 & 1 & 0 \\ 2 & 1 & -2 & 0 & 0 & 1 \end{array}\right] \begin{array}{l} \\ R_2 + 3R_1 \\ R_3 - 2R_1 \end{array}$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 2 & 0 \\ 0 & -1 & 3 & 3 & 7 & 0 \\ 0 & 1 & -2 & -2 & -4 & 1 \end{array}\right] \begin{array}{l} \\ R_3 + R_2 \\ \\ \end{array} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 2 & 0 \\ 0 & -1 & 3 & 3 & 7 & 0 \\ 0 & 0 & 1 & 1 & 3 & 1 \end{array}\right] \begin{array}{l} \\ -R_2 \\ \\ \end{array}$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 2 & 0 \\ 0 & 1 & -3 & -3 & -7 & 0 \\ 0 & 0 & 1 & 1 & 3 & 1 \end{array}\right] \begin{array}{l} \\ R_2 + 3R_3 \\ \\ \end{array} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 2 & 0 \\ 0 & 1 & -3 & 0 & 2 & 3 \\ 0 & 0 & 1 & 1 & 3 & 1 \end{array}\right]$$

Thus, $A^{-1} = \begin{bmatrix} 1 & 2 & 0 \\ 0 & 2 & 3 \\ 1 & 3 & 1 \end{bmatrix}$

2. Let $M = \begin{bmatrix} 1 & 1 \\ -1 & -1 \end{bmatrix}$, $N = \begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix}$ and $P = \begin{bmatrix} -1 & -1 \\ 3 & 4 \end{bmatrix}$.
Compute M^2, MN, MP

Solution:
$M^2 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

$MN = \begin{bmatrix} 2 & 3 \\ -2 & -3 \end{bmatrix}$

$MP = \begin{bmatrix} 2 & 3 \\ -2 & -3 \end{bmatrix}$

3. Let A, B and C be a 2×2 matrices. Show that the following statements are false.
(a) If $A^2 = 0$ then $A = 0$.

Solution: This statement is false. For example, when $A = \begin{bmatrix} 1 & 1 \\ -1 & -1 \end{bmatrix}$, from question 2, we have $A^2 = 0$ but $A \neq 0$.
(b) If $AB = AC$ and $A \neq 0$ then $B = C$.

Solution: This statement is false. For example, when $A = M$, $B = N$ and $C = P$ as in question 2, we have $AB = AC$ and $A \neq 0$, but $B \neq C$.

QUIZ 2

Thursday, January 11, 2010 at 17:00 **Duration: 30 minutes.**

1. Let $A = \begin{bmatrix} 1 & 2 & 0 \\ 0 & 2 & 3 \\ 1 & 3 & 1 \end{bmatrix}$. Find A^{-1}. Make sure that you show all the work.

Solution:

$$\left[\begin{array}{ccc|ccc} 1 & 2 & 0 & 1 & 0 & 0 \\ 0 & 2 & 3 & 0 & 1 & 0 \\ 1 & 3 & 1 & 0 & 0 & 1 \end{array}\right] \begin{array}{l} \\ \\ R_3 - R_1 \end{array} \left[\begin{array}{ccc|ccc} 1 & 2 & 0 & 1 & 0 & 0 \\ 0 & 2 & 3 & 0 & 1 & 0 \\ 0 & 1 & 1 & -1 & 0 & 1 \end{array}\right] \begin{array}{l} R_1 - 2R_3 \\ R_2 - 2R_1 \\ \\ \end{array}$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & -2 & 3 & 0 & -2 \\ 0 & 0 & 1 & 2 & 1 & -2 \\ 0 & 1 & 1 & -1 & 0 & 1 \end{array}\right] \begin{array}{l} R_1 + 2R_2 \\ \\ R_3 - R_2 \end{array} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 7 & 2 & -6 \\ 0 & 0 & 1 & 2 & 1 & -2 \\ 0 & 1 & 0 & -3 & -1 & 3 \end{array}\right] R_2 \leftrightarrow R_3$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 7 & 2 & -6 \\ 0 & 1 & 0 & -3 & -1 & 3 \\ 0 & 0 & 1 & 2 & 1 & -2 \end{array}\right]$$

Thus, $A^{-1} = \begin{bmatrix} 7 & 2 & -6 \\ -3 & -1 & 3 \\ 2 & 1 & -2 \end{bmatrix}$

2. Let $M = \begin{bmatrix} -1 & -1 \\ 1 & 1 \end{bmatrix}$, $N = \begin{bmatrix} \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} \end{bmatrix}$ and $P = \begin{bmatrix} -1 & -1 \\ 3 & 4 \end{bmatrix}$.

Compute M^2, MN, MP

$M^2 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

$MN = \begin{bmatrix} -1 & -1 \\ 1 & 1 \end{bmatrix} = M$, and

$MP = \begin{bmatrix} -2 & -3 \\ 2 & 3 \end{bmatrix}$

3. Let A, B and C be a 2×2 matrices. Show that the following statements are false.

(a) If $A \neq 0$ then $A^2 \neq 0$.

Solution: This statement is false. For example, when $A = \begin{bmatrix} -1 & -1 \\ 1 & 1 \end{bmatrix}$, we have $A \neq 0$, but from question 2, $A^2 = 0$.

(b) If $AB = A$ then $B = I$.

Solution: This statement is false. For example, when $A = M$ and $B = N$ as in question 2, we have $AB = A$, but $A^2 \neq I$.